CS 584/684 Algorithm Design and Analysis

Homework 5

Portland State U, Winter 2021 — *02/09/21*

Lecturer: Fang Song — *Due: 02/16/21*

Instructions. This problem set contains 5 pages (including this cover page) and 3 questions. A random subset of problems will be graded.

- Your solutions will be graded on *correctness* and *clarity*. You should only submit work that you believe to be correct, and you will get significantly more partial credit if you clearly identify the gap(s) in your solution. It is good practice to start any long solution with an informal (but accurate) summary that describes the main idea. You may opt for the "I take 15%" option.
- You need to submit a PDF file before the deadline. Either a clear scan of you handwriting or a typeset document is accepted. You will get 5 bonus points for typing in LaTeX (Download and use the accompany TeX file).
- You may collaborate with others on this problem set. However, you must **write up your own solutions** and **list your collaborators and any external sources** for each problem. Be ready to explain your solutions orally to a course staff if asked.
- For problems that require you to provide an algorithm, you must give a precise description of the algorithm, together with a proof of correctness and an analysis of its running time. You may use algorithms from class as subroutines. You may also use any facts that we proved in class or from the book.

1. (15 points) Suppose you are given a set L of n line segments in the plane, where each segment has one endpoint on the vertical line $x = 0$ and one endpoint on the vertical line $x = 1$, and all $2n$ endpoints are distinct. Describe an algorithm to compute the largest subset of L in which no pair of segments intersects.

2. (Shortest path with bounded negative edges) Suppose we are given a directed graph G with weighted edges and two vertices s and t.

   (a) (15 points) Describe an algorithm to find the shortest path from s to t when exactly one edge in G has negative weight.

(b) (10 points (bonus)) Describe an algorithm to find the shortest path from s to t when exactly k edges in G have negative weights. How does the running time of your algorithm depend on k?

3. (Arbitrage) Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example (exchange rates not up to date), suppose 1 US dollar buys 71 Indian rupees, 1 Indian rupee buys 1.6 Japanese yen, and 1 Japanese yen buys 0.0093 US dollars. Then by converting currencies, a trader can start with 1 US dollar and buy $71 \times 1.6 \times 0.0093 = 1.0565$ US dollars, thus making a profit of 5.65 percent.

   Suppose that you are given n currencies $c_1, c_2, \ldots, c_n$ and an $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i,j]$ units of currency j.

   Describe an algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i_1}, \ldots, c_{i_k} \rangle$ such that

   $$R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1] > 1\,.$$

   You may assume that there is an algorithm to detect negative-weight cycles in a graph running in time $O(mn)$.