

Winter 2018 CS 485/585 Introduction to Cryptography

LECTURE 15

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*Agenda*

- (Last time) PKE
- Digital signature INTRO
- Review of HW3
- Quiz3

*Defining digital signatures*

**Definition 1** (KL-12.1). A digital signature scheme consists PPT algorithms  $(G, S, V)$  such that:

1.  $G: (pk, sk) \leftarrow G(1^n)$ .
2.  $S$ : on input  $sk$  and message  $m$ , outputs  $\sigma \leftarrow S_{sk}(m)$ .
3.  $V$ : on input  $pk$  and message-signature pair  $(m, \sigma)$ , output  $V_{pk}(m, \sigma) = \text{acc/rej}$ .

*Comparison to MAC.* DS and MAC both protect data integrity. One drawback of DS, as in PubKE, is that it is usually less efficient than MAC. Otherwise DS are advantageous in many aspects.

- No need to share a private-key with each user. Everyone just generates its  $(pk_i, sk_i)$  pair and makes  $pk_i$  public.
- Publicly verifiable and transferable.
- Non-repudiation. Once a sender signs a message, s/he cannot later deny having done so.

*A common misconception.* Signature is often considered the “inverse” of public-key encryption. Use decryption as signing, and encrypting as verification. This is *totally unsound!*

*Software update example* How to use a signature scheme?

- MS generates  $(pk, sk)$ . Publish  $pk$ .
- Software patch  $m$  is signed under  $sk$ :  $\sigma \leftarrow S_{sk}(m)$ .
- User downloads  $(m, \sigma)$ , and verifies  $\sigma$  with  $pk$ ,  $V_{pk}(m, \sigma) = b$ . If  $b = \text{acc}$ , execute  $m$  and complete update.

*Security of Digital signature.* Intuitively, we would need that no adversary without knowing the secret key can produce valid signatures. The formal definition is similar to that of MAC, considering chosen-message-attacks where an adversary may ask to see signatures on messages of its choice.

**FS NOTE:** Draw sig-forge game

1.  $CH$  generates  $(pk, sk) \leftarrow G(1^n)$ .
2.  $\mathcal{A}$  is given  $pk$  and access to signing oracle  $S_{sk}(\cdot)$ . Let  $\mathcal{L} := \{m_i\}$  be the set of messages that  $\mathcal{A}$  has queried. At the end  $\mathcal{A}$  outputs  $(m^*, \sigma^*)$ .
3. We say that  $\mathcal{A}$  succeeds if 1)  $V_{pk}(m^*, \sigma^*) = \text{acc}$ ; and 2)  $m^* \notin \mathcal{L}$ . Define the output of the game  $\text{Sig-forge}_{\mathcal{A}, \Pi}(n) = 1$  iff.  $\mathcal{A}$  succeeds.

Figure 1: Signature forgery game  $\text{Sig-forge}_{\mathcal{A}, \Pi}(n)$

**Definition 2** ([KL: Def. 12.2]). A signature scheme  $\Pi = (G, S, V)$  is existentially unforgeable under an adaptive chosen-message attack (eu-cma-secure), if for all PPT  $\mathcal{A}$ ,

$$\Pr[\text{Sig-forge}_{\mathcal{A}, \Pi}(n) = 1] \leq \text{negl}(n).$$

We will see constructions next time.

### Hash-and-Sign

Suppose that we already have a secure signature scheme that can sign messages of fixed length  $\ell(n)$ . How to sign longer messages? Hash function will save our day, as in hash-and-MAC.

Let  $\Pi = (G, S, V)$  be a signature scheme for messages of length  $\ell(n)$ , and let  $H : \{0, 1\}^* \rightarrow \{0, 1\}^{\ell(n)}$  be hash function. Construct a new signature scheme  $\Pi' = (G', S', V')$

1.  $G' = G$ :  $(pk, sk) \leftarrow G(1^n)$ .
2.  $S'$ : on input message  $m \in \{0, 1\}^*$ , output  $\sigma \leftarrow S_{sk}(H(m))$ .
3.  $V'$ : accept  $(m, \sigma)$  iff.  $V_{pk}(H(m), \sigma) = 1$ .

**Theorem 3** ([KL: Thm. 12.4]). *If  $\Pi$  is a secure signature and  $H$  a collision resistant hash, then construction above  $\Pi'$  is a secure signature (for arbitrary-length messages).*

Review HW3

Note that implicitly,  $\mathcal{A}$  also has access to the verification procedure  $V_{pk}(\cdot)$ .