CS 410/510 Introduction to Quantum Computing

Homework 3

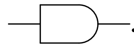
Portland State U, Spring 2020	*04/19/2020*
Lecturer: Fang Song	*Due: 11:59pm PDT, 04/26/2020*

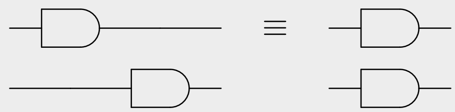
Instructions. This problem set contains 9 pages (including this cover page) and 3 questions. Problems marked with "[**G**]" are required for 510 students. Students enrolled in 410 will get bonus points for solving them. A random subset of problems will be graded.

Your solutions will be graded on *correctness* and *clarity*. You should only submit work that you believe to be correct, and you will get significantly more partial credit if you clearly identify the gap(s) in your solution. It is good practice to start any long solution with an informal (but accurate) summary that describes the main idea.

You need to submit a PDF file via Gradescope before the deadline. Either a clear scan of you handwriting or a typeset document is accepted. You will get 5 bonus points for typing in LaTeX (Download and use the accompany TeX file).

You may collaborate with others on this problem set. However, you must ***write up your own solutions*** and ***list your collaborators*** for each problem.

1. (6 points) (Partial measurement) Given two qubits A and B in an arbitrary state $|\psi\rangle_{AB}$, show that measuring A and then B is *equivalent* to measuring both qubits simultaneously. Namely the two quantum circuits below are equivalent. All measurements are in the computational basis denoted by .



2. (Distinguishing states by local measurements) Suppose Alice and Bob are physically separated from each other, and are each given one of the qubits of some 2-qubit state. They are required to distinguish between State I and State II with only local measurements. Namely they can each perform a local (one-qubit) unitary operation and then a measurement (in the computational basis) of their own qubit. After their measurements, they can send only classical bits to each other. (This is usually referred to as LOCC: local operation and classical communication.) In each case below, either give a perfect distinguishing procedure (that never errs) or explain why there is no perfect distinguishing procedure (i.e., that for any procedure the success probability must be less than 1).

   (a) (6 points) State I: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$; State II: $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

(b) (6 points) State I: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$; State II: $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

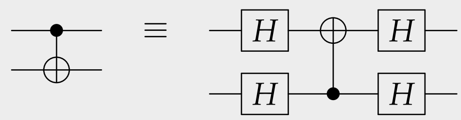
(c) (6 points) **[G]** State I: $\frac{1}{\sqrt{2}}(|00\rangle + i|11\rangle)$; State II: $\frac{1}{\sqrt{2}}(|00\rangle - i|11\rangle)$

3. (Hadamard) Let H be the Hadamard gate.

   (a) (6 points) Let $a, b \in \{0, 1\}$ be two arbitrary bits. Show that $H(\frac{|0\rangle + (-1)^{a \oplus b}|1\rangle}{\sqrt{2}}) = |a \oplus b\rangle$.

(b) (6 points) Show that $\forall x \in \{0,1\}^n, H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x \cdot y} |y\rangle$. Here $x \cdot y = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n \bmod 2$.

(c) (6 points) Verify the following circuit identity. (Hint: it suffices to check that they act the same on the computational basis.)



(d) (8 points) Continue from part (c). Implement the two circuits with complete measurement in the end in IBM Qiskit (via either graphic composer, notbook, or local Qiskit environment). Run the QASM simulator 1024 shots with each of the Bell states as input, and show the output histograms.