CS 410/510 Introduction to Quantum Computing

Homework 2

Portland State U, Spring 2020 *04/12/2020*

Lecturer: Fang Song *Due: 11:59pm PDT, 04/19/2020*

Instructions. This problem set contains 9 pages (including this cover page) and 3 questions. Problems marked with "[**G**]" are required for 510 students. Students enrolled in 410 will get bonus points for solving them. A random subset of problems will be graded.

Your solutions will be graded on *correctness* and *clarity*. You should only submit work that you believe to be correct, and you will get significantly more partial credit if you clearly identify the gap(s) in your solution. It is good practice to start any long solution with an informal (but accurate) summary that describes the main idea.

You need to submit a PDF file via Gradescope before the deadline. Either a clear scan of you handwriting or a typeset document is accepted. You will get 5 bonus points for typing in LaTeX (Download and use the accompany TeX file).

You may collaborate with others on this problem set. However, you must ***write up your own solutions*** and ***list your collaborators*** for each problem.

Across this entire problem set, let

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

1. (Tensor product) Recall the *tensor product* of two matrices A and B is $A \otimes B := (a_{ij}B)$.

    (a) (6 points) Write out the 4×4 matrix representing $X \otimes Y$. Does it equal $Y \otimes X$?

(b) (6 points) Show that if U and V are unitary matrices, then $U \otimes V$ is also unitary.

(c) (6 points) Let A, B, C and D be matrices such that the matrix products AC and BD are well defined. Show that $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

2. (Quantum states and gates)

   (a) (12 points) For each of the processes below, describe the resulting quantum states.

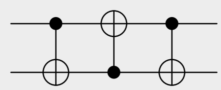
      i) Apply H to the first qubit of state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

      ii) Apply H to both qubits of state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

      iii) Apply $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ to both qubits of state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

(b) (6 points) Suppose we have two qubits. We apply X to both and then Z to both. Is it equivalent to applying Z to both and then applying X to both? Determine your answer by explicitly computing $X \otimes X$, $Z \otimes Z$, and their products both ways.

(c) (6 points) Analyze the following quantum circuit. Describe its effect and write down its matrix representation.



3. (Product states versus entangled states) In each of the following, either express the 2-qubit state as a tensor product of 1-qubit states or prove that it cannot be expressed this way.

   (a) (5 points) $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

(b) (5 points) **[G]** $\frac{3}{4}|00\rangle + \frac{\sqrt{3}}{4}|01\rangle + \frac{\sqrt{3}}{4}|10\rangle + \frac{1}{4}|11\rangle$