Portland State U

F, 04/17/2020

S′20 CS 410/510

Intro to quantum computing

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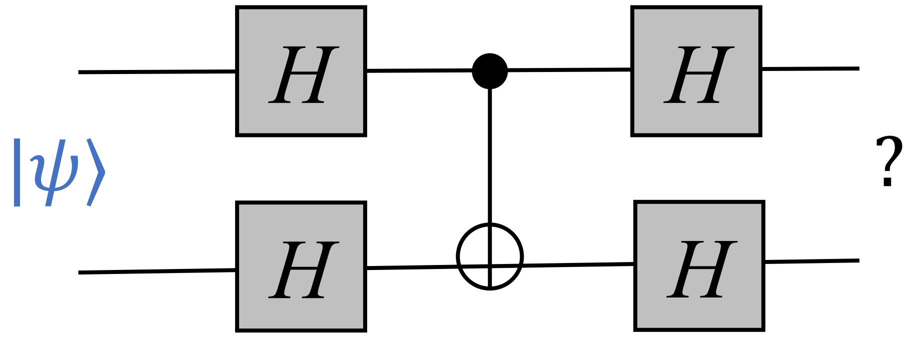
Week 3

- Quantum postulates
- Distinguishing quantum states
- Deutsch / Deutsch-Josza algorithms

Credit: based on slides by Richard Cleve

Exercise



Input	Output
$\vert 00\rangle$	?
$\vert 01\rangle$	?
$\vert 00\rangle$	?
$\vert 01\rangle$	?

Logistics

- HW2 due Sunday

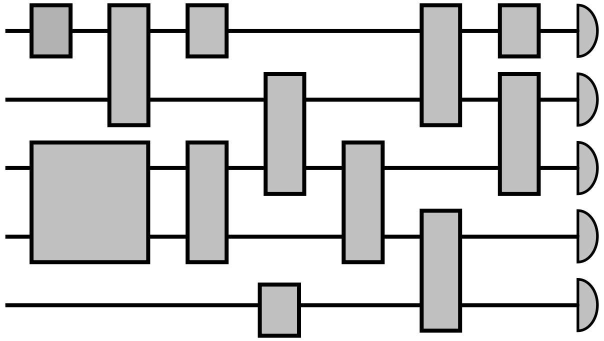
- Remarks
  - Campuswire: support markdown and LaTeX (e.g., `$$e^{iH}$$` = e^{iH});
  - Campuswire: stay informed, settings → notifications → digest messages
  - Youtube playlist: read the description (e.g., time stamps)

- Project: discussion at end of class

Postulates of quantum theory

1. States
2. Operations (dynamics)
3. Measurement
4. Composite systems

Quantum circuit model
(quantum computer)



Postulate 1: quantum states

- n-qubit system ⇔ (Hilbert) state space: $\mathbb{C}^{2^n} = (\mathbb{C}^2)^{\otimes n}$
- Computational (standard) basis: $\{|x\rangle : x \in \{0,1\}^n\}$
- Quantum state: 2^n-dim. unit vector

$$\forall x \in \{0,1\}^n, \alpha_x \in \mathbb{C}, \sum_x |\alpha_x|^2 = 1$$

$$\begin{pmatrix} \alpha_{000} \\ \alpha_{001} \\ \alpha_{010} \\ \alpha_{011} \\ \alpha_{100} \\ \alpha_{101} \\ \alpha_{110} \\ \alpha_{111} \end{pmatrix} = \sum_{x\in\{0,1\}^3} \alpha_x |x\rangle$$

$$|000\rangle = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} |001\rangle = \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} |010\rangle = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} |011\rangle = \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix}$$

$$|100\rangle = \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} |101\rangle = \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} |110\rangle = \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} |111\rangle = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix}$$

$n = 3$

Postulate 2: operations

- System evolution ⇔ Unitary transformation $|\psi_1\rangle = U|\psi_0\rangle$
- If you are really curious of the physics:

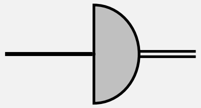
H: Hamiltonian of the system, a Hermitian matrix $(H = H^\dagger)$

Schrodinger's equation: $i\frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$

→ $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$. $U := e^{-iHt}$ Unitary.

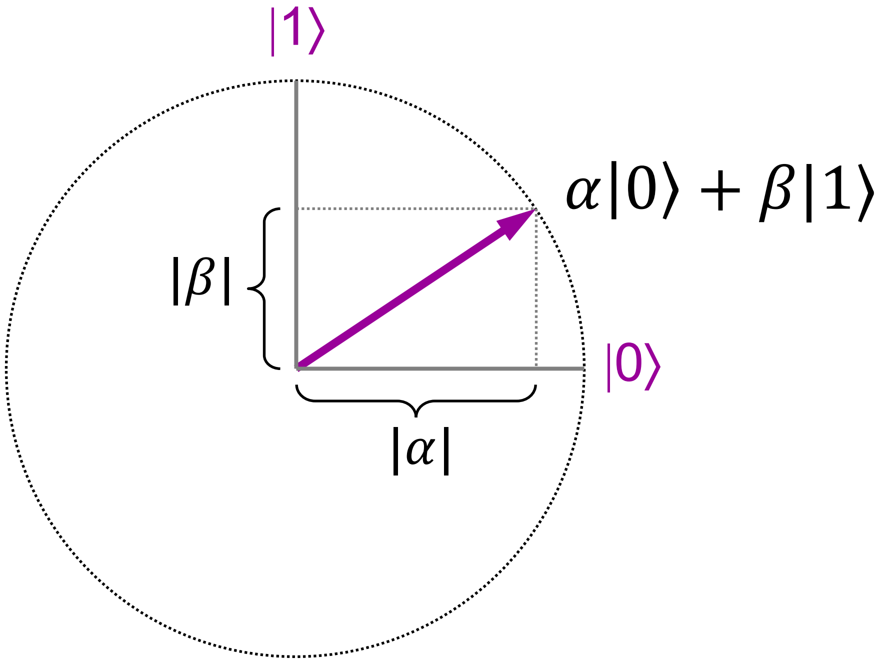
Postulate 3: measurements

- Standard measurement (in computational basis)

$\|\psi\rangle = \sum_{x\in\{0,1\}^n} \alpha_x \|x\rangle$		See	w. probability	posterior state
		x	$\|\alpha_x\|^2$	$\|x\rangle$

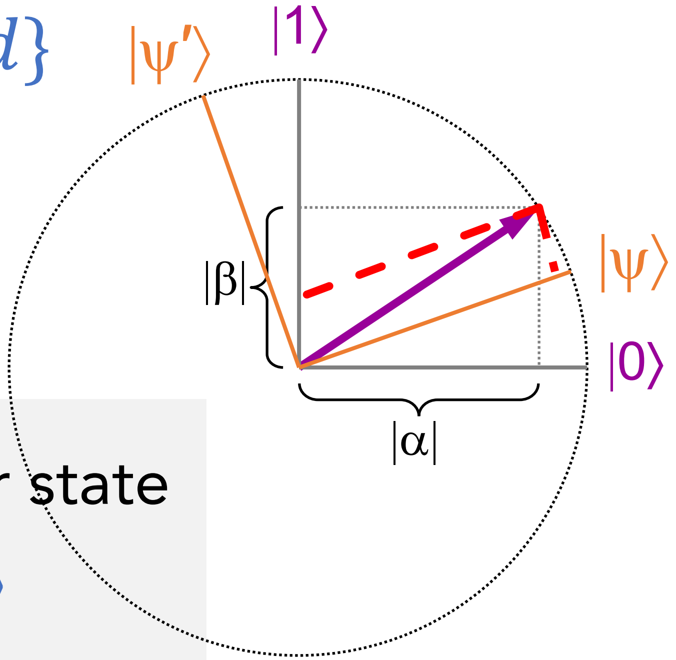
- Geometric picture: projection

$$\Pr(\text{observe } x) = |\alpha_x|^2 = |\langle x|\psi\rangle|^2$$



Measuring in an orthonormal basis

- Recall: orthonormal basis of $\mathbb{C}^d$ $\{|e_j\rangle : j = 1, \ldots, d\}$
  - $\forall j, \| |e_j\rangle \| = 1$
  - $\forall i \neq j, \langle e_i | e_j \rangle = 0$



	See	w/ probability	posterior state
Measure $|\psi\rangle$ in $\{|e_j\rangle\}$ $\mapsto$	j	$|\langle e_j|\psi\rangle|^2$	$|e_j\rangle$

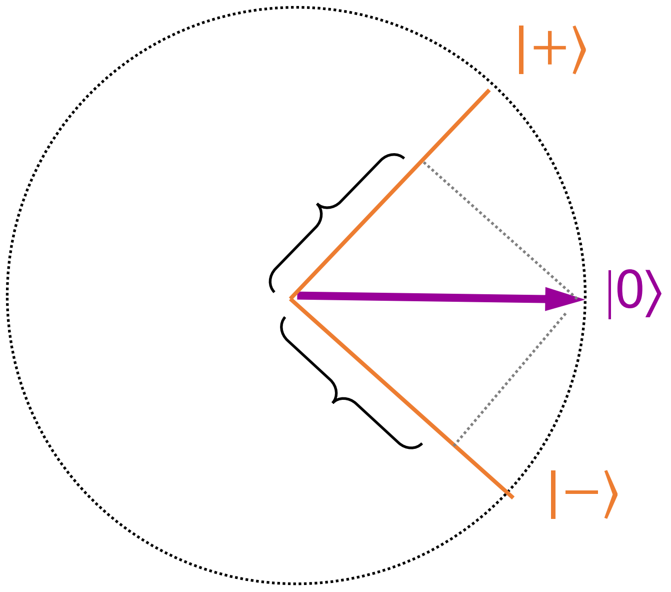
j is merely a label of $|e_j\rangle$

Measuring in an orthonormal basis

	See	w/ probability	posterior state
Measure $\|\psi\rangle$ in $\{\|e_j\rangle\}$ $\mapsto$	j	$\|\langle e_j\|\psi\rangle\|^2$	$\|e_j\rangle$

Ex. Measure in $\{|+\rangle, |-\rangle\}$

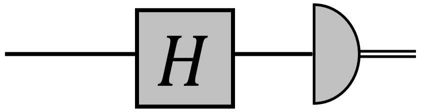
	See	w/ probability	posterior state
$\|+\rangle \mapsto$	$+$	$\|\langle +\|+\rangle\|^2 = 1$	$\|+\rangle$
	$-$	$\|\langle -\|+\rangle\|^2 = 0$	$\|-\rangle$
$\|0\rangle \mapsto$	$+$	$\|\langle +\|0\rangle\|^2 = 1/2$	$\|+\rangle$
	$-$	$\|\langle -\|0\rangle\|^2 = 1/2$	$\|-\rangle$



Implement measurement in arb. basis

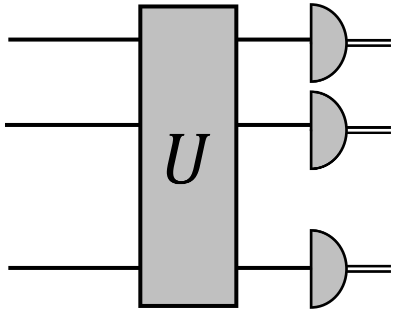
Theorem. Meas. in any $\{|e_j\rangle\} \equiv$ Unitary + standard meas.

- Measure in $\{|+\rangle, |-\rangle\}$



$$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \overset{?}{\rightarrow} \alpha|0\rangle + \beta|1\rangle$$

- General case: measure in $\{|e_j\rangle\}$



$$U: |e_j\rangle \mapsto |j\rangle \quad U = \sum_j |j\rangle\langle e_j|$$

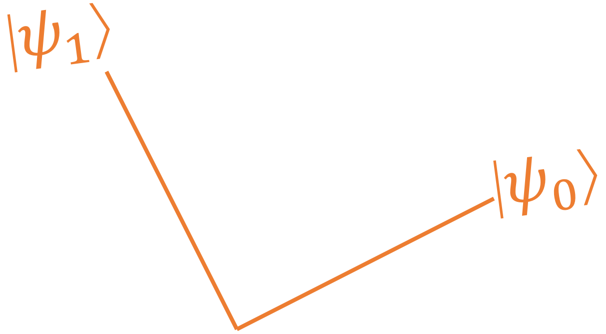
Distinguishing quantum states

Cor. Orthogonal quantum states can be distinguished perfectly

- Given a qubit $|\phi\rangle \in \{|\psi_0\rangle, |\psi_1\rangle\}$ w. $\langle\psi_0|\psi_1\rangle = 0$

? $|\psi_0\rangle$ or $|\psi_1\rangle$

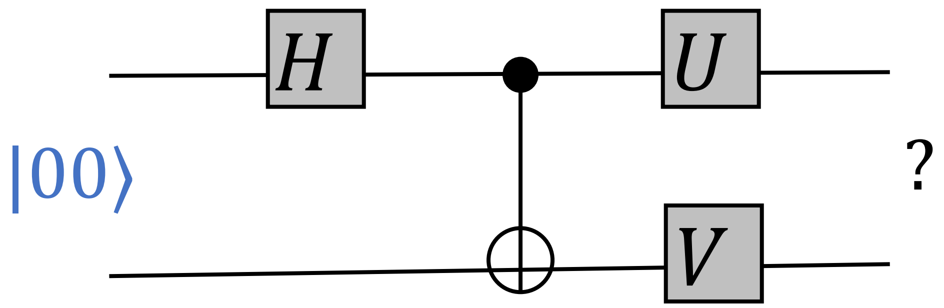
! Measure in $\{|\psi_0\rangle, |\psi_1\rangle\}$



- Given $|\phi\rangle \in |\psi_1\rangle, \dots, |\psi_k\rangle \in \mathbb{C}^d, k \le d.\ \forall i \ne j, \langle\psi_j|\psi_i\rangle = 0$
  - Complete $|\psi_1\rangle, \dots, |\psi_k\rangle$ to an orthonormal basis $\{|\psi_j\rangle : j = 1, \dots, d\}$
  - Measure $|\phi\rangle$ in $\{|\psi_j\rangle : j = 1, \dots, d\}$

Exercise

1



U	V	Output
I	I	?
X	I	?
I	Z	?
Z	Z	?

Hint: you will get Bell states

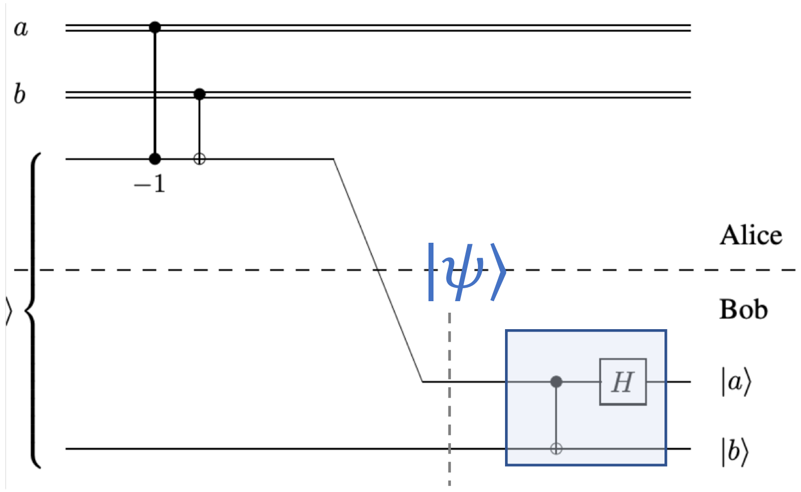
Exercise

2 Show that the 4 states form an orthonormal basis for 2 qubits

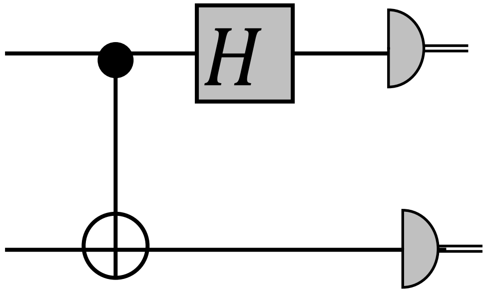
Exercise

3. Design a circuit to implement the measurement in this basis

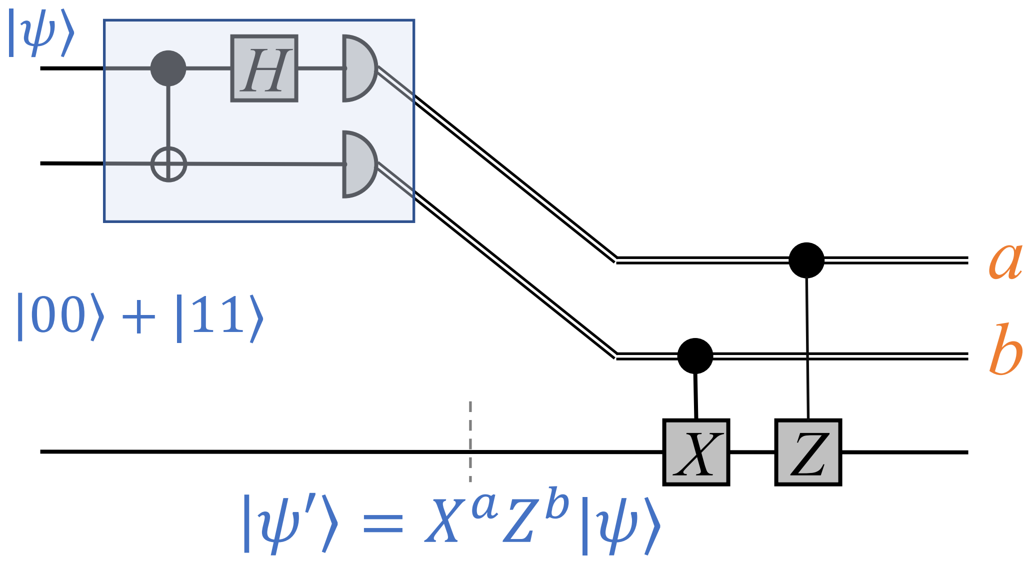
Reflection on superdense coding & teleportation



ab	$\|\psi\rangle$
00	$\|00\rangle + \|11\rangle$
01	$\|10\rangle + \|01\rangle$
10	$\|00\rangle - \|11\rangle$
11	$\|10\rangle - \|01\rangle$



Distinguish Bell states

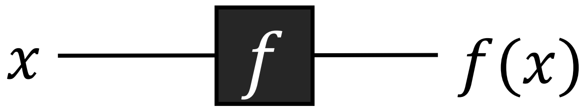


Ex. What is the state of the top 2 qubits after the measurement?

Deutsch's algorithm

Black-box function and query model

Given: a function f as a black box (a.k.a. oracle)



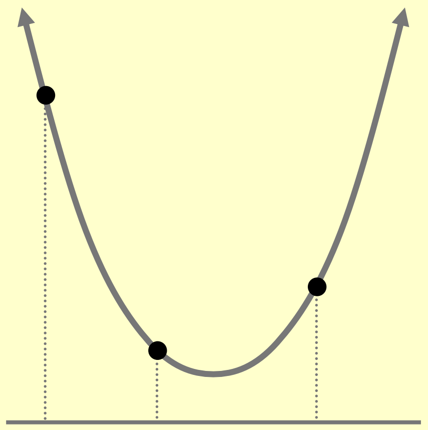
Goal: determine some property about f making as few queries to f (and other operations) as possible

Example. Polynomial interpolation

Let: $f(x) = c_0 + c_1 x + c_2 x^2 + \ldots + c_d x^d$

Goal: determine $c_0, c_1, c_2, \ldots, c_d$

Question: How many f-queries does one require for this?

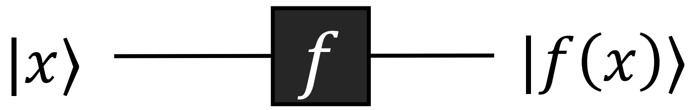


Answer: $d + 1$

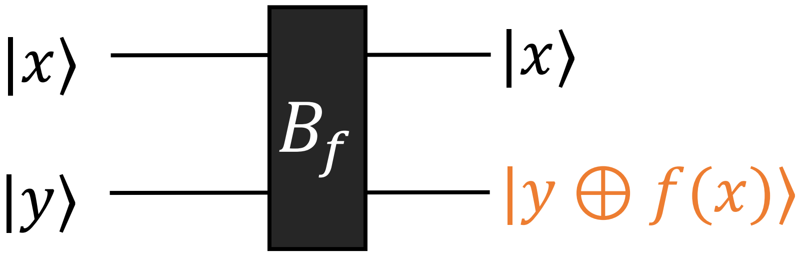
Quantum black-box function

Quantum operations need to be unitary (reversible)

☹ Not reversible



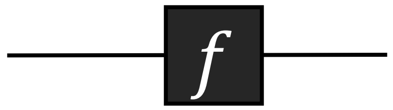
☺ Unitary



Can query in superposition

Deustch's problem

Let $f: \{0,1\} \rightarrow \{0,1\}$



There are four possibilities:

x	$f_1(x)$
0	0
1	0

x	$f_2(x)$
0	1
1	1

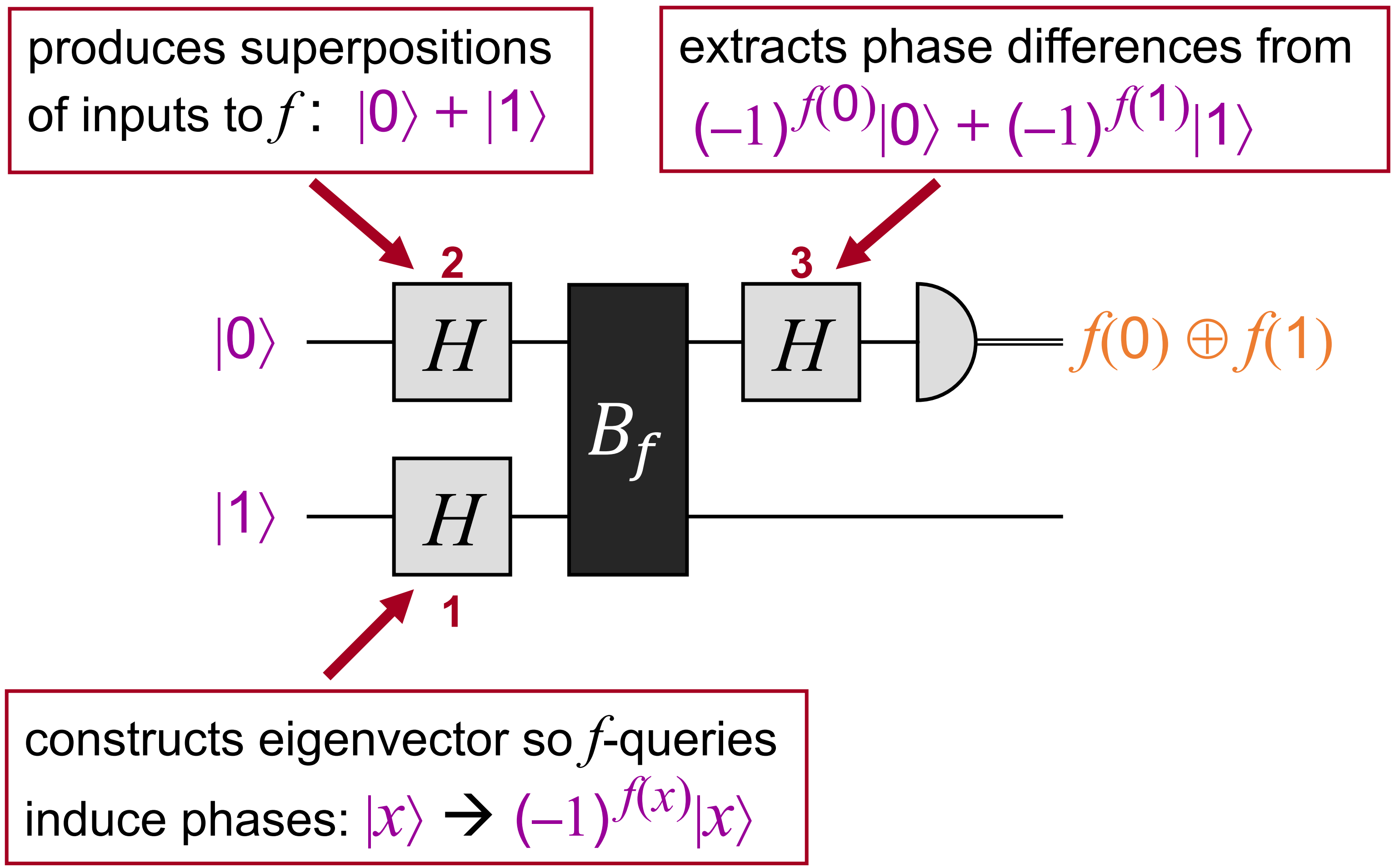
x	$f_3(x)$
0	0
1	1

x	$f_4(x)$
0	1
1	0

Goal: determine whether or not $f(0) = f(1)$ (i.e. $f(0) \oplus f(1)$)

- Any classical method requires ***two*** queries
- What about a quantum method?

Summary of Deutsch's algorithm



Deutsch-Josza algorithm

Deutsch-Josza problem

Let $f: \{0,1\}^n \rightarrow \{0,1\}$ be either constant or balanced, where

- ***constant*** means $f(x) = 0$ for all x, or $f(x) = 1$ for all x
- ***balanced*** means $\sum_x f(x) = 2^{n-1}$

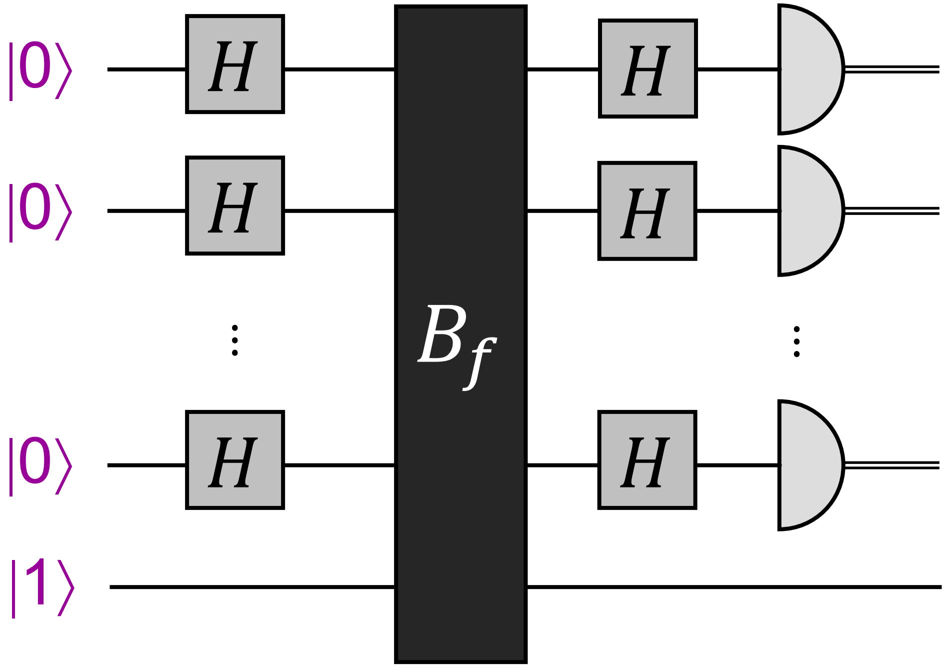
Goal: determine whether f is constant or balanced

- How many queries are there needed classically?

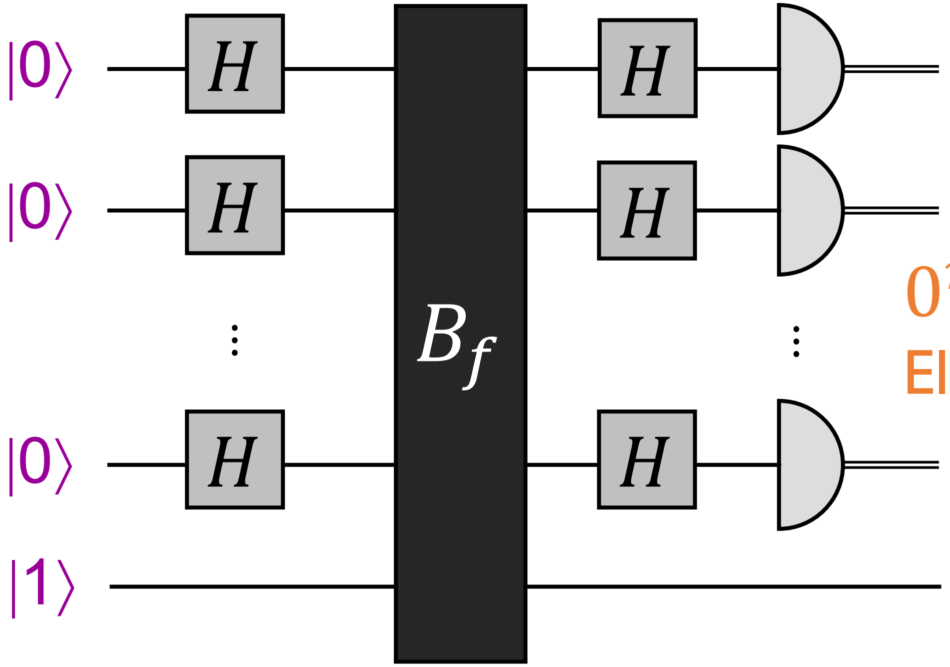
  Deterministic algorithms. ___

  Randomized algorithms. ___

Deutsch-Josza quantum algorithm



Summary of Deutsch-Josza algorithm



0^n: constant

Else: balanced

Scratch