Portland State U

Spring'20 CS 410/510

Intro to quantum computing

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Agenda

Warmup

- COVID-19 response
  - Take it seriously; health is 1st priority; ask for help;
- Zoom: rules; handsup: virtually, physically; add to calendar;
- Recording disclaimer
- Class intro

Tools

- Coursewebpage
- D2L: quizzes / grades etc.
- Campuswire: 0460
- Gradescape

Upload a profile picture!

This class

Goal

- Go over syllabus: know a new paradigm; practice analytical skills; horn critical thinking (be an educated audience)

Format

- Flipped, to accommodate your schedule; but you have to do your work

Policy

- Grading: Quiz+HW+Project+Participation
- HW collaboration: as much as you can
- HW writup: on your own; LaTeX (reward bonus point)

Questions?

Break 3 mins

Quantum basics

What is computation?

- Classical vs. Quantum computers

Quantum basics

- Qubit, single-qubit gate, measurement,

Computation:

? why can we do what we do on a computer

↓

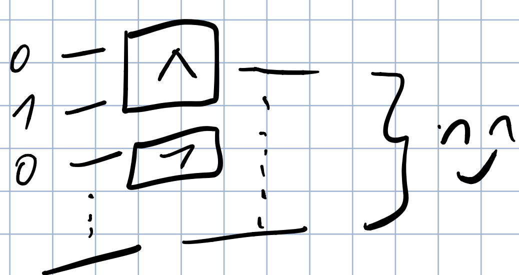
what are the foundamental laws: what's possible & NOT?

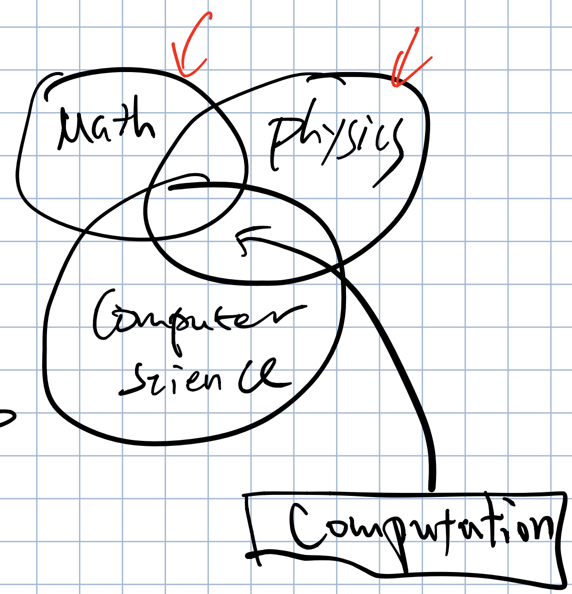
→ physics!

we don't have to worry about physics

→ Math: a formal computational model

- Turing Machine
- Boolean ckt





What if we are unhappy w/ existing computers?

a. Are there better Models [under the same physical laws]?

:(ECCT

b. Better physics laws?

:) Quantum physics

↙

A new type of computer possible.

Quantum Computer

1. QC is extradinary!

2. QC is NOT grand/Mystic!

* Can QC be built, ever?

  - Breakthrough in rezent years
    [Google, IBM . - - -]
    Quantum Supremacy

  - A valid computation model
    under Q physics

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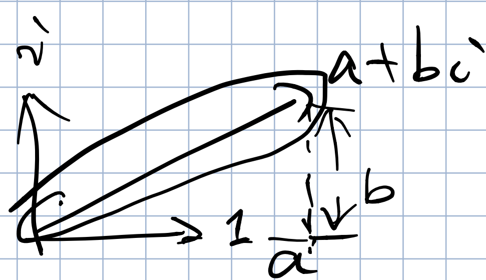
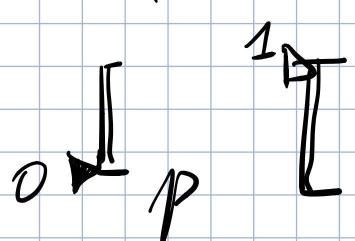
2. Quantum Basics.

- Quantum physics is "extended" prob. theory.



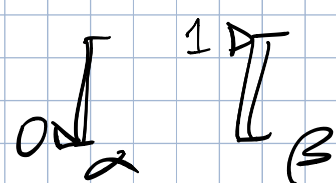
Probability

- BIT



- $0 \le p, q \le 1$
  $p + q = 1$ $\quad \begin{pmatrix} p \\ 1-p \end{pmatrix}$

Quantum BIT (Qubit)



(α : magnitude
$\alpha = a + bi$
$|\alpha|^2 = a^2 + b^2$

- amplitudes
  $\alpha, \beta \in \mathbb{C}$
  $|\alpha|^2 + |\beta|^2 = 1$ $\quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$
  Quantum Superposition

- BIT

Quantum BITs.

· operators.

· BIT FLIP

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} \mapsto \begin{pmatrix} 1-p \\ p \end{pmatrix}$$

· General op's:

stochastic matrix

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad a_i \geq 0$$

$$+ \quad = 1 \quad 1$$

preserves 1-Norm of the prob. vector.

U: preserve the 2-Norm ?

$$U\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1 \qquad |\alpha'|^2 + |\beta'|^2 = 1$$

· Quantum op's.

· $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$X\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

· $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$H\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\left|\frac{1}{\sqrt{2}}\right|^2 + \left|\left(-\frac{1}{\sqrt{2}}\right)\right|^2 \overset{?}{=} 1 \checkmark$$

· General:

unitary matrices

$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad a = x + yi$

$UU^{\dagger} = \mathbb{1}$ $\quad a^* = x - yi$

"dagger" → $U^{\dagger} = (U^*)^T$

$$= \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix}^T = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}$$

Diraz notation "ket"

$$|\phi\rangle = \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = (\alpha^* \ \beta^*)\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

"Bra"

$$\langle\phi| := (\alpha^* \ \beta^*)$$

$$= \langle\phi|\phi\rangle$$

$$|\phi'\rangle = U|\phi\rangle$$

$$\langle\phi'|\phi'\rangle = \langle\phi|U^\dagger U|\phi\rangle$$

row Veztor, matrix, col Vez.

$$= \langle\phi|\mathbb{1}|\phi\rangle$$

$$= \langle\phi|\phi\rangle$$

- A spezial operation.

You flip a COIN $\begin{pmatrix}p \\ 1-p\end{pmatrix}$

Take a look (observe)

→ either "0" : p

OR "1" : $1-p$

Quantum superposition

$$\alpha|0\rangle + \beta|1\rangle$$

- "Observe" a Qubit

Quantum meas.

$\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ "collapse" post-meas state

$\xrightarrow{\text{meas}}$ "obs" w.p. $|0\rangle$

"0" $|\alpha|^2$ $\begin{pmatrix}1 \\ 0\end{pmatrix}$

"1" $|\beta|^2$ $\begin{pmatrix}0 \\ 1\end{pmatrix}$

$$\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \alpha\begin{pmatrix}1 \\ 0\end{pmatrix} + \beta\begin{pmatrix}0 \\ 1\end{pmatrix}$$

Exer

Your questions
More to work on in groups

1. Let . What is ?
2. Let . What is ?
3. Measurement

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\rightarrow |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

outer product

$tr(A|+\rangle\langle+|)$

1. $A\,|+\rangle\langle+|$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a+b & a+b \\ c+d & c+d \end{pmatrix}$

$tr(A|+\rangle\langle+|)$

$= \frac{1}{2}(a+b+c+d)$

Claim: $Tr(ABC) = Tr(CBA) = Tr(ACB)$

$Tr(A|1\rangle\langle 0|)$

$= Tr(\langle 0|A|1\rangle)$

$= b$

$Tr(A|0\rangle\langle 1|) = Tr(\langle 1|A|0\rangle)$

$= c$