CS 410/510 Introduction to Quantum Computing

Lecture 6

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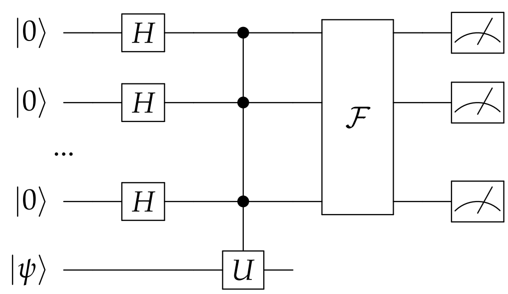
1 Phase Estimation

Remember, for this problem, our input is a quantum circuit for a unitary U, as well as the eigenvector $|\psi\rangle$. $U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$. Our goal is to approximate θ.



1.1 Special Case

First, lets consider the special case $\theta = \frac{j}{2^m}$. Remember, the state going into F is

$$\frac{1}{\sqrt{2^m}} \sum_{x\in\{0,1\}^m} e^{2\pi i\theta x}|x\rangle$$

Let $\omega := e^{\frac{2\pi i}{2^m}}$, rewrite it as $\frac{1}{\sqrt{2^m}} \sum_{x\in\{0,1\}^m} \omega^{jx}|x\rangle =: |\phi_j\rangle$. Therefore, computing j amounts to distinguishing each $|\phi_j\rangle$.

Now, consider $\{|\phi_j\rangle : j \in \{0,1\}^m\}$. $\langle\phi_j|\phi'_{j'}\rangle$ will equal 1 if $j = j'$, and 0 if otherwise. This forma an orthonormal basis for m-qubit states in $\mathbb{C}$. There is also a standard basis $\{|j\rangle : j \in \{0,1\}^m\}$. We know that there exists a unitary F such that $|\phi_j\rangle \overset{F}{\mapsto} |j\rangle$. So, if we apply F to $|\phi_j\rangle$, we will see j.

But what is F? If $j = 0$, that means F is the following $2^m \times 2^m$ matrix.

$$F = \begin{bmatrix} 1 & 1 & \dots & 1 & \dots & 1 \\ 1 & \omega & \dots & \omega^j & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & \omega^{2^m-1} & \dots & \omega^{j(2^m-1)} & \dots & \omega^{(2^m-1)(2^m-1)} \end{bmatrix}$$

This is the discrete Fourier Transform.

1.2 General Case

Now, lets look at the general case, where $\theta \in [0, 1)$. We will see the following.

$$F|x\rangle = \frac{1}{\sqrt{2^m}} \sum_{y \in \{0,1\}^m} \omega^{-yx} |y\rangle$$

$$F|\phi\rangle = \frac{1}{\sqrt{2^m}} \sum_{x \in \{0,1\}^m} \omega^{jx} F|x\rangle$$

$$F|\phi\rangle = \frac{1}{2^m} \sum_{x \in \{0,1\}^m} \omega^{jx} \sum_{y \in \{0,1\}^m} \omega^{-yx} |y\rangle$$

$$F|\phi\rangle = \frac{1}{2^m} \sum_{y \in \{0,1\}^m} (\sum_{x \in \{0,1\}^m} e^{2\pi i x(\theta - \frac{y}{2^m})}) |y\rangle$$

For measuring, we will see this.

$$Pr[y] = \left| \frac{1}{2^m} \sum_{x \in \{0,1\}^m} e^{2\pi i x(\theta - \frac{y}{2^m})} \right|^2$$

Note that this looks exactly like a geometric series. That means we can rewrite it without the $\sum$.

$$Pr[y] = \frac{1}{2^{2m}} | \frac{e^{2\pi i (M\theta - y)} - 1}{e^{2\pi i (\theta - \frac{y}{M})} - 1} |^2$$

[Fang: add the details of analysis]

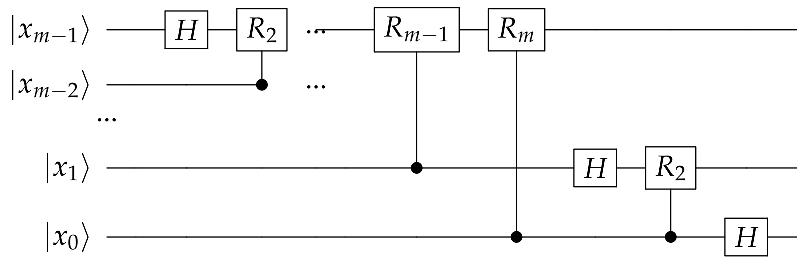
This means $\theta = \frac{j}{M} + \epsilon$, $|\epsilon| \leq \frac{1}{2^{m+1}}$. Essentially, this ends up showing $Pr[j] \geq \frac{4}{\pi^2} \geq 0.4$. The larger Pr[j] becomes, the more precise our approximation of θ will be. In short, the chance that we will have m bits corrext at least 40% of the time.

2 Quantum Fourier Transform

[Fang: More to add here]

The following shows a quantum circuit for computing Fourier Transform.

$$R_k = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^k}} \end{bmatrix}$$



Lastly, the outputs are all reversed such that $|x_0\rangle$ is on the top.