CSCE629 Analysis of Algorithms

Homework 10

Texas A&M U, Fall 2019 — *11/08/19*

Lecturer: Fang Song — *Due: 10am, 11/15/19*

Instructions.

- Typeset your submission by LaTeX, and submit in PDF format. Your solutions will be graded on *correctness* and *clarity*. You should only submit work that you believe to be correct, and you will get significantly more partial credit if you clearly identify the gap(s) in your solution. You may opt for the "I'll take 15%" option.
- You may collaborate with others on this problem set. However, you must **write up your own solutions** and **list your collaborators and any external sources** for each problem. Be ready to explain your solutions orally to a course staff if asked.
- For problems that require you to provide an algorithm, you must give a precise description of the algorithm, together with a proof of correctness and an analysis of its running time. You may use algorithms from class as subroutines. You may also use any facts that we proved in class or from the book.
- If you describe a Greedy algorithm, you will get no credit without a formal proof of correctness, even if your algorithm is correct.

This assignment contains 3 questions, 4 pages for the total of 55 points and 0 bonus points. A random subset of the problems will be graded.

1. (Integer linear programming) An *integer linear-programming* problem is a linear-programming problem with the additional constraint that the variables x must take on *integral* values.

   (a) (7 points) Show that *weak duality* (CLRS Lemma 29.8) holds for an integer linear program.

   (b) (8 points) Show that *strong duality* (CLRS Theorem 29.10) does not always hold for an integer linear program.

   (c) (10 points) Given a primal linear program in standard form, let us define P to be the optimal objective value for the primal linear program, D to be the optimal objective value for its dual, IP to be the optimal objective value for the integer version of the primal (that is, the primal with the added constraint that the variables take on integer values), and ID to be the optimal objective value for the integer version of the dual. Assuming that both the primal integer program and the dual integer program are feasible and bounded, show that

$$IP \leq P = D \leq ID \,.$$

2. (NP under regular operations) Given any sets $X \subseteq \{0,1\}^*$ and $Y \subseteq \{0,1\}^*$, we can define new sets under regular operations. For example, the *union* is $X \cup Y = \{x : x \in X \text{ or } x \in Y\}$; and the *concatenation* is $X \circ Y = \{xy : x \in X \text{ and } y \in Y\}$. We consider applying these operations on problems in NP.

   (a) (7 points) Prove or disprove that NP is closed under union. Namely for any $X \in \mathrm{NP}$ and $Y \in \mathrm{NP}$, does $X \cup Y \in \mathrm{NP}$ always hold?

   (b) (8 points) Prove or disprove that NP is closed under concatenation.

   (c) (Exercise. Do not turn in.) Prove or disprove that NP is closed under complement.

3. (15 points) (2-SAT) Consider 2-SAT, in which the input is a formula with at most 2 literals per clause. Show that 2-SAT is in P. (Hint: you may want to consider an algorithm for testing if a graph is bipartite).