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Zero-knowledge proof systems for QuantumMA

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How does cryptography **change** in a quantum world?

- Quantum attacks

Hard problems broken
- Factoring & DL [Shor'94],
- Some lattice problems [EHKS'14,BS'16,CDPR'16]

Security analyses fail
- Unique quantum attacks arise
- Difficult to reason about quantum adversaries!

- Quantum protocols

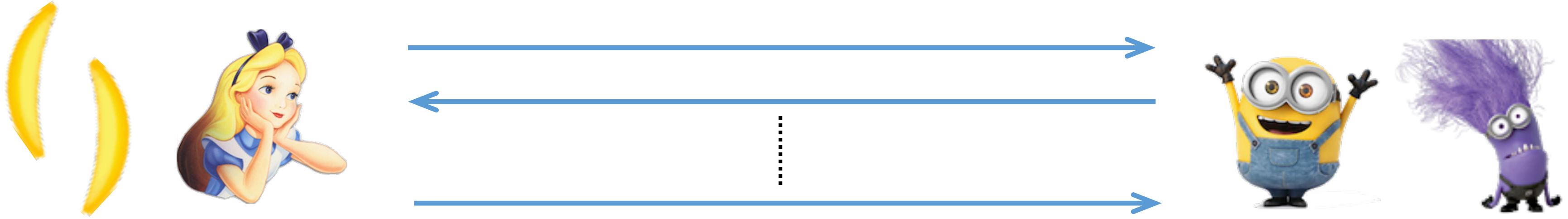
Outperform classical protocols
- Ex. Quantum key distribution

Crypto tools for quantum tasks
- Ex. Encrypt quantum data

Today's Topic

Zero-Knowledge proof systems

[GoldwasserMicaliRacoff STOC'84]



The two bananas can be transformed into each other

I'm convinced!
But I still don't know how

What problems can be proven in Zero-Knowledge?

Today in history: ZK for NP

What problems can be proven in Zero-Knowledge?

[GoldreichMicaliWidgerson FOCS'86]

Every problem in **NP** has a zero-knowledge proof system*

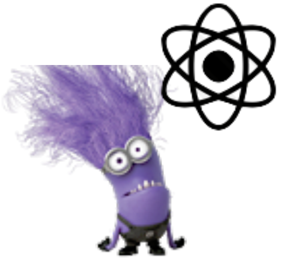
* Under suitable hardness assumptions

- Invaluable in modern cryptography

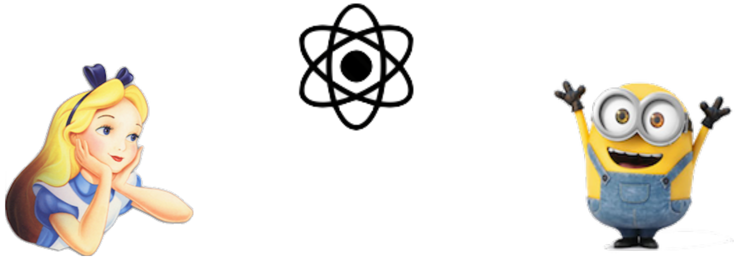
Today: ZK in a quantum world

What problems can be proven in Zero-Knowledge *quantumly*?

1. Do classical protocols remain Zero-Knowledge against quantum malicious verifiers?



2. Can honest users empower quantum capability and prove problems concerning quantum computation?



ZK in a quantum world: status

1. Classical ZK against quantum attacks: big challenge

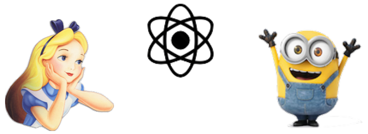
- **Rewinding**: difficult against quantum attackers [Graaf'97]

  Critical for showing ZK classically

- Special quantum rewinding [Watrous'06]
  - GMW protocol can be made quantum-secure
  - many other cases not applicable

⇐ Quantum-secure ✔ ZK for NP ⇒

2. ZK proofs for quantum problems: little known

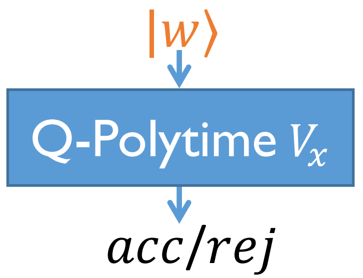


Our main result

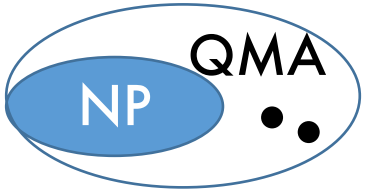
Every problem in QMA has a zero-knowledge proof system*

quantum analogue of NP (MA)

- Problems *verifiable* by efficient **quantum** alg.



- Power: $\exists\ L$ in QMA NOT believed in NP (ex. group non-membership)



- Nice features of our construction:
  - Simple structure 3-“move”: commit-challenge-respond
  - All communication classical except first message
  - (Almost) minimal assumption: same as GMW with quantum resistance
  - Efficient prover: useful to build larger crypto constructions

Our additional contributions

New tools for quantum crypto and quantum complexity theory

- Proposing a new complete problem for QMA

  Corollary: QMA = QMA with very limited verifier

  - Simpler proof than some recent work [MorimaeNF'15'16]

Further implications?

- A quantum encoding mechanism, supporting
  - "somewhat homomorphic"
  - Perfect secrecy
  - Authentication

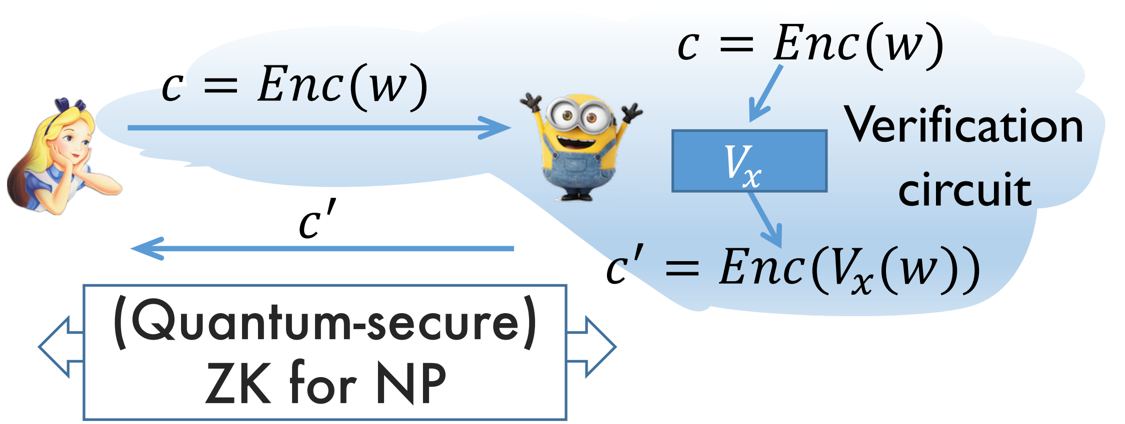
Other applications?

Our construction:
ZK for QMA

Inspiration: ZK by homomorphic encryption

Reductionist's wishful thinking:
reduce (ZK for QMA) to (ZK for NP)

- **I seem to know how to:** reduce (ZK for NP) to (ZK for NP)



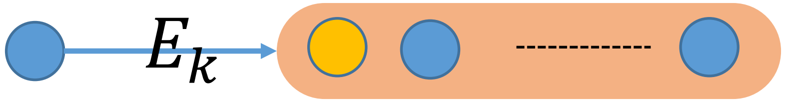
- Decode of c' is *accept*

- Verifier homomorphically evaluates Verification ckt
- Prover proves in ZK: the result encodes "*accept*"

- **Challenges of adapting to QMA:**
  - Right tools in the quantum setting: encoding, etc?
  - How to prevent dishonest verifier?

Evaluate another circuit compute 1^{st} bit of w!

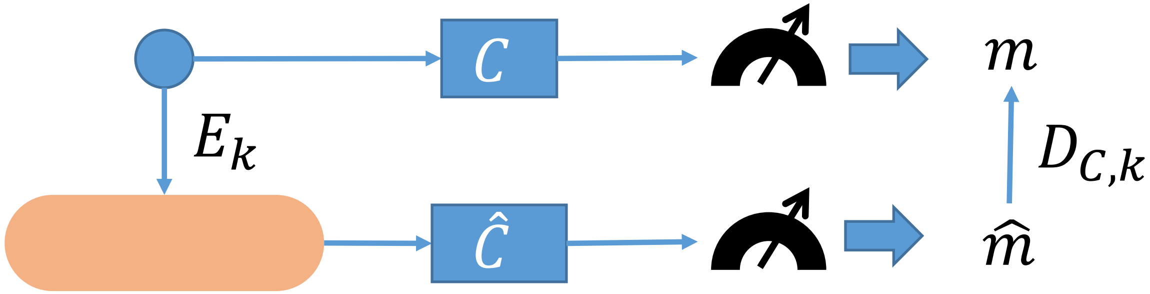
Build quantum tool I: a new encoding scheme



* Based on quantum error correcting & (trap) quantum auth. scheme [BGS12]

- Augmented trap scheme*, supporting

i. Clifford circuits $\mathcal{C}$ & measure, transversally (“somewhat homomorphic”)



ii. Perfect secrecy

iii. **Authentication**

- Dishonest behavior can be detected

- But: verification of existing QMA-complete problems require more than $\mathcal{C}$

$\mathcal{C}$: simple, non-universal

Build quantum tool II: a new QMA-complete problem

- Local Clifford-Hamiltonian (**LCH**) Problem

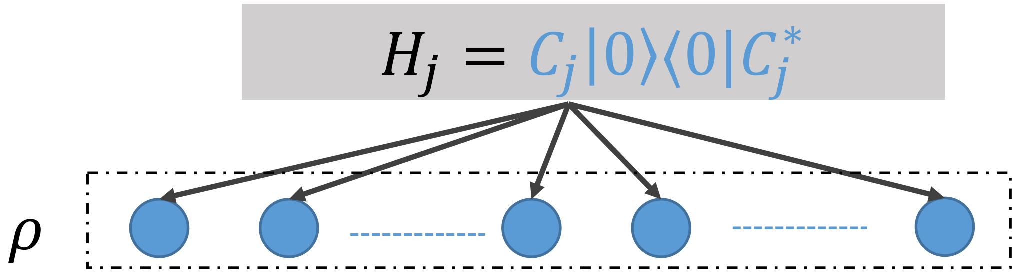
Verification circuit
- Pick small random part of witness
- Apply Clifford $C \in \mathcal{C}$ &measure:
  - non-zero string → accept

Can run **Verification** on *encoded* witness (by AugTrap) transversally

Input: Hamiltonian operators $H_1, \ldots H_m$, each H_j on 5 qubits & of form $C_j|0\rangle\langle 0|C_j^*$

- **YES**: $\exists$ n-qubit state ρ, $\langle \rho, \sum H_j \rangle \leq 2^{-n}$ (no violation, low eigenvalue)
- **NO**: $\forall$ n-qubit state ρ, $\langle \rho, \sum H_j \rangle \geq 1/n$ (lots violation, large eigenvalue)

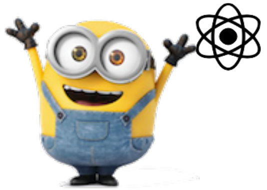
$C_j \in \mathcal{C}$ Clifford



ZK proof system for LCH

Input: $H_1, \ldots, H_m,\ H_j = C_j\ |0\rangle\langle 0| C_j^*$

Witness $|w\rangle$



1 AugTrap Enc w. key k $|w\rangle$ Committing k k

2 Pick random j and measure H_j on encoded witness, outcome $\hat{m}_j$

$j, \hat{m}_j$

3 Check $\hat{m}_j$ & H_j consistent (i.e. verifier was honest)

4 Open key bits k_j of term j

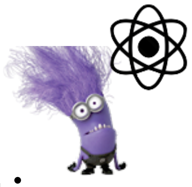
5 Invoke quantum-secure ZK proof for NP

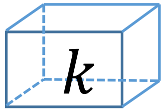
a. open/commit consistent
b. decoding $\hat{m}_j$ according to key k_j not violate H_j

- Nice features
  - Simple structure 3-“move”
  - All but first message classical
  - Efficient prover
  - Only assuming: commitment (to classical msg) that is quantum-secure

Our ZK protocol for LCH works

- Completeness: ✔
- Soundness: ✔
  - Full proof non-trivial, relying on error correcting code & binding of commit
- Zero-knowledge: for any malicious verifier



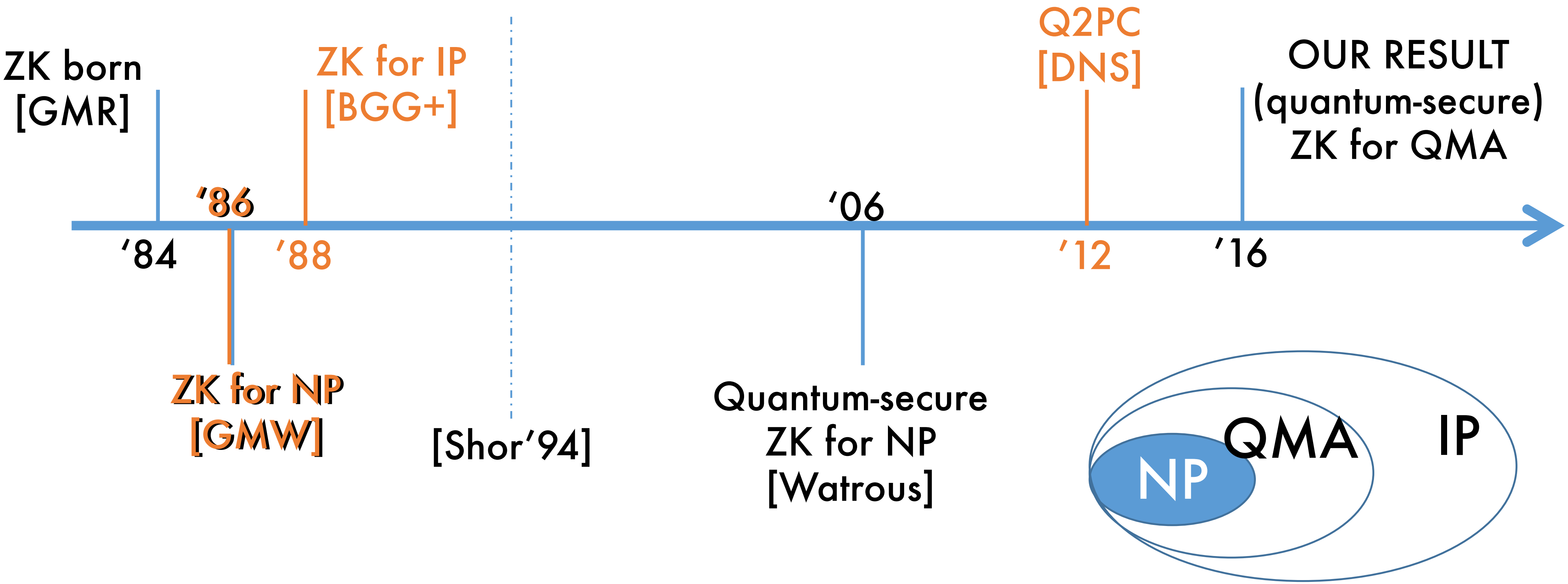
$E_k(|w\rangle)$ + 

Can be viewed as hybrid encryption

  - Verifier's measurement produces classical encrypted msg
  - "Leakage" resilient: k_j doesn't compromise secrecy on remaining qubits

Corollary: any problem in QMA has a ZK proof system

Timeline in retrospect: alternate approaches?



Comparison

	GMW analogue[1]	ZK for IP[1]	Q2PC[1]	Our protocol
All QMA	✗	✔	✔	✔
Prover efficiency	✔	✗	✔	✔
Mild assumption[2]	✔	✔	✗	✔
Round #	✔	✗	✗[3]	✔
Availability	✔	✔✔[4]	✗	✔

1. plausible, but needs double-check; 2. commitment vs. dense PKE
3. depends on V's ckt; 4. purely classical

Concluding Remarks

Every QMA problem has a "nice" zero-knowledge proof system

New tools for quantum crypto & quantum complexity theory

- QMA complete: local Clifford Hamiltonian Problem
- Augmented Trap encoding scheme

- Open Questions

1. ZK for QMA

- purely classical protocol (w. efficient prover)?
- constant-round (CR) w. negl. soundness error:
  - CRZK for NP (Q-Security unknown) ➔ CRZK for QMA

2. Proof of *quantum* knowledge?

3. QPIP

- verifying a quantum computer by a classical computer

Thank you!