CS 410/510 Introduction to Quantum Computing

Lecture 11

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1 Theory of Quantum Information

A.K.A. Quantum information processing

- *What it's not*: Quantum computation
  - Algorithms (e.g., search, order-finding)
  - Complexity (e.g., QBP, QMA)
  - In general: *Making quantum states meaningful.*
- *What it is*: Quantum information
  - More fundamental tasks (e.g., create, copy, store, communicate, ...)
  - Focus on theory (experiments also exist)
  - In general: *Making quantum states availiable.*

2 Information theory

2.1 Classical setting

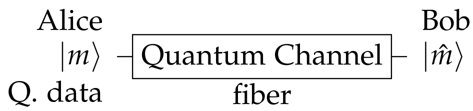
Claude Shannon, "A Mathematical Theory of Communication," 1948.

Alice Bob
m — Comm. Channel — $\hat{m}$
phone line, cable, etc.

Central Questions:

0. What is information?
1. How many bits do we need to describe a source? (Shannon entropy) [Shannon's Source Coding Theorem]
2. How can we transmit messages over a noisy channel? [Shannon's Noisy-channel Coding Theorem]

2.2 Quantum setting



Central questions:

	classical channel	quantum channel
classical data	1) Source Coding Thm. 2) Noisy Channel Thm.	1) How much classical info is in a Q. state [Holevo] 2) Classical capacity of Q. channel (QECC)
quantum data		1) Min. qubits to describe Q. source [Schumacher] 2) Q. capacity of Q. channel (QECC)

New resource: Entanglement

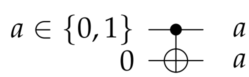
- Teleportation: Transmit quantum states over a classical using two bits
- Superdense coding: Transmit two classical bits using a single qubit
- Non-local games

New tasks:

- Getting around the No-cloning Theorem
- Distinguishing quantum states

3 No-cloning Theorem

Classically...



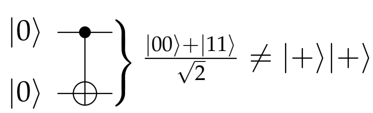
The above classical circuit "clones" the bit a reversibly. I.e., it calculates $\rangle a, 0\langle \mapsto \rangle a, a\langle$.

Quantumly...

Is there a quantum circuit U that calculates $|a\rangle|0\rangle \overset{U}{\mapsto} |a\rangle|a\rangle$? *No-cloning Theorem*

Why does CNOT fail?
Succeeds in some cases, e.g., $|0\rangle, |1\rangle$:

Fails in others, e.g., $|+\rangle$:



Proof. Suppose for contradiction that there exists a unitary U and a fixed preparation $|s\rangle$ s.t. for all states $|\psi\rangle$,

$$|\psi\rangle|s\rangle \overset{U}{\mapsto} |\psi\rangle|\psi\rangle$$

Then for any distinct quantum states $|\psi\rangle$ and $|\psi'\rangle$,

$$|\psi\rangle|s\rangle \overset{U}{\mapsto} |\psi\rangle|\psi\rangle$$
$$|\psi'\rangle|s\rangle \overset{U}{\mapsto} |\psi'\rangle|\psi'\rangle$$

And since unitary operations preserve inner product,

$$\langle \psi s|\psi' s\rangle = \langle \psi\psi|\psi'\psi'\rangle \quad (1)$$
$$(\langle\psi| \otimes \langle s|)(|\psi'\rangle \otimes |s\rangle) = (\langle\psi| \otimes \langle\psi|)(|\psi'\rangle \otimes |\psi'\rangle) \quad (2)$$
$$\langle\psi|\psi'\rangle \otimes \langle s|s\rangle = \langle\psi|\psi'\rangle \otimes \langle\psi|\psi'\rangle \quad (3)$$
$$\langle\psi|\psi'\rangle = \langle s|s\rangle \quad (4)$$

And since $\langle s|s\rangle$ is a constant, line 4 clearly cannot be true for all distinct $|\psi\rangle$, $|\psi'\rangle$. Therefore by contradiction, no such U can exist. □

4 The Power of Entanglement

Examples of entangled states:

- 2-qubit

$$\text{Bell States: } \begin{cases} \left.\begin{aligned} |\Phi^+\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\ |\Phi^-\rangle &= \tfrac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \end{aligned}\right\} \text{EPR Pairs} \\ |\Psi^+\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\ |\Psi^-\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \end{cases}$$

- 3-qubit

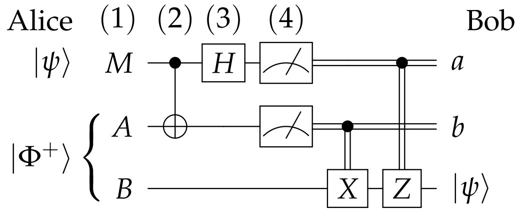
$$\text{GHZ State: } \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

Teleportation:
Suppose Alice wants to send $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob.

- Alice may not know α and β.
- Even if she does, she still cant send $\alpha, \beta \in \mathbb{C}$ with infinite precision.

Claim: If Alice and Bob share an EPR pair, Alice can send 2 classical bits to Bob s.t. Bob can reproduce $|\psi\rangle$ exactly [Bennet, Brassard, et al. '93].

Teleportation circuit:



1. $\frac{1}{\sqrt{2}}(\alpha(|000\rangle + |011\rangle) + \beta(|100\rangle + |111\rangle))$
2. $\frac{1}{\sqrt{2}}(\alpha(|000\rangle + |011\rangle) + \beta(|110\rangle + |101\rangle))$
3. $\frac{1}{2}(|00\rangle(\alpha|0\rangle + \beta|1\rangle)|10\rangle(\alpha|0\rangle - \beta|1\rangle)|01\rangle(\alpha|1\rangle + \beta|0\rangle)|11\rangle(\alpha|1\rangle - \beta|0\rangle))$
4.

MA	w.p.	B		
00	$\frac{1}{4}$	$\alpha\|0\rangle + \beta\|1\rangle$	$\xmapsto{\mathbb{1}}$	$\|\psi\rangle$
01	$\frac{1}{4}$	$\alpha\|1\rangle + \beta\|0\rangle$	$\xmapsto{X}$	$\|\psi\rangle$
10	$\frac{1}{4}$	$\alpha\|0\rangle - \beta\|1\rangle$	$\xmapsto{Z}$	$\|\psi\rangle$
11	$\frac{1}{4}$	$\alpha\|1\rangle - \beta\|0\rangle$	$\xmapsto{ZX}$	$\|\psi\rangle$