Classical Cryptographic Protocols in a Quantum World

Fang Song

Joint work with Sean Hallgren and Adam Smith

Computer Science and Engineering
Penn State University

Are classical cryptographic protocols secure against quantum attackers?

Are classical cryptographic protocols secure against quantum attackers?

- Some protocols: no longer secure
  - Computational assumptions broken by efficient quantum alg's
    - Factoring and Discrete Logarithm [Shor'94]
    - Principal ideal problem [Hallgren'02]
  - Information-theoretical classically secure protocol also broken
    - A two prover commitment scheme becomes non-binding [Crepeau,Salvail,Simard,Tapp'06]
    - Attackers only need storing entanglement
- Many protocols: unknown how to prove security
  - Classical proof techniques may no longer apply: e.g. rewinding
  - General question: how to reason about quantum adversaries?

Classical Protocols Secure against Quantum Attacks

- Some tasks are achievable
  - Zero-Knowledge (ZK) for NP [Watrous'09]
    - Quantum rewinding in a special case
  - ZK for a larger class of languages [Hallgren,Kolla,Sen,Zhang'08]
  - Coin-flipping [Damgaard,Lunemann'09]
  - Proofs of knowledge (PoK) [Unruh'10]

Question: using classical protocols, is *every* task achievable against classical attackers also achievable against quantum attackers?

a. proving security of existing protocols

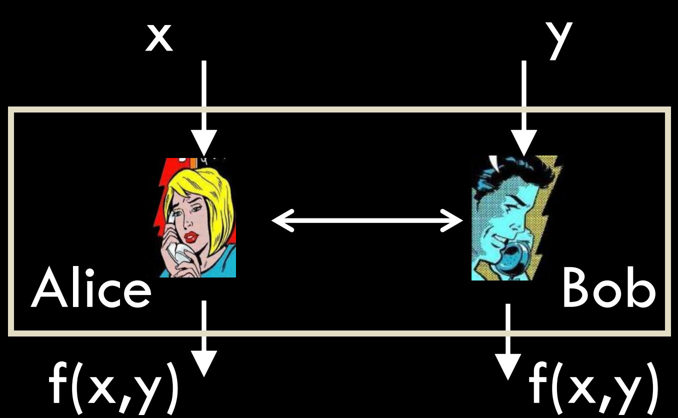
b. designing new protocols

Our Contribution

Main Result:

∃ **classical** *secure function evaluation* protocols against **quantum** attacks

Parallels classical feasibility results: [Yao'86;Goldreich,Micali,Wigderson'87]



Secure Function Evaluation (SFE)

- Correctness: Jointly evaluate f(x,y) correctly
- Privacy: Bob does not learn anything about x beyond f(x,y); same for Alice

Our Contribution

Main Result:

∃ **classical** *secure function evaluation* protocols against **quantum** attacks

Parallels classical feasibility results: [Yao'86;Goldreich,Micali,Wigderson'87]

a. Prove a family of classical arguments goes through against quantum adversaries

- Corollary: *fully simulatable* **ZKPoK** ⇨ **quantum**-secure SFE

b. Construct a fully simulatable **ZKPoK** against quantum adv's

- Get around difficulty of quantum rewinding

- Revisit quantum stand-alone security models (see paper)

Building SFE from ZKPoK

- Identify a family of *hybrid arguments* that goes through against quantum adv's

D ≈ D' ≈ D'' ≈ - · - · - ≈ E

  - Adjacent pairs only differs by "simple" changes:
    - E.g., changing the plaintext of an encryption
  - Formalize a **Simple Hybrid Argument** framework
    - Resembles code-based games [Bellare,Rogaway'06]
- A classical construction [Canetti,Lindell,Ostrovsky,Sahai'02] fits SHA framework
  - [CLOS'02]: fully simulatable ZKPoK ⇨ classically secure SFE
- Corollary: fully simulatable ZKPoK ⇨ **quantum**-secure SFE, assuming
  - Quantum-secure dense encryption & pseudorandom generators
  - Implied by, e.g, Learning-with-errors (LWE) assumption

Our Contribution

Main Result:

∃ classical *secure function evaluation* protocols against **quantum** attacks

Parallels classical feasibility results: [Yao'86;Goldreich,Micali,Wigderson'87]

a. Prove a family of classical arguments goes through against quantum adversaries

- Corollary: *Fully simulatable* **ZKPoK** ⇨ **quantum** secure SFE

b. Construct a fully simulatable **ZKPoK** against quantum adv's

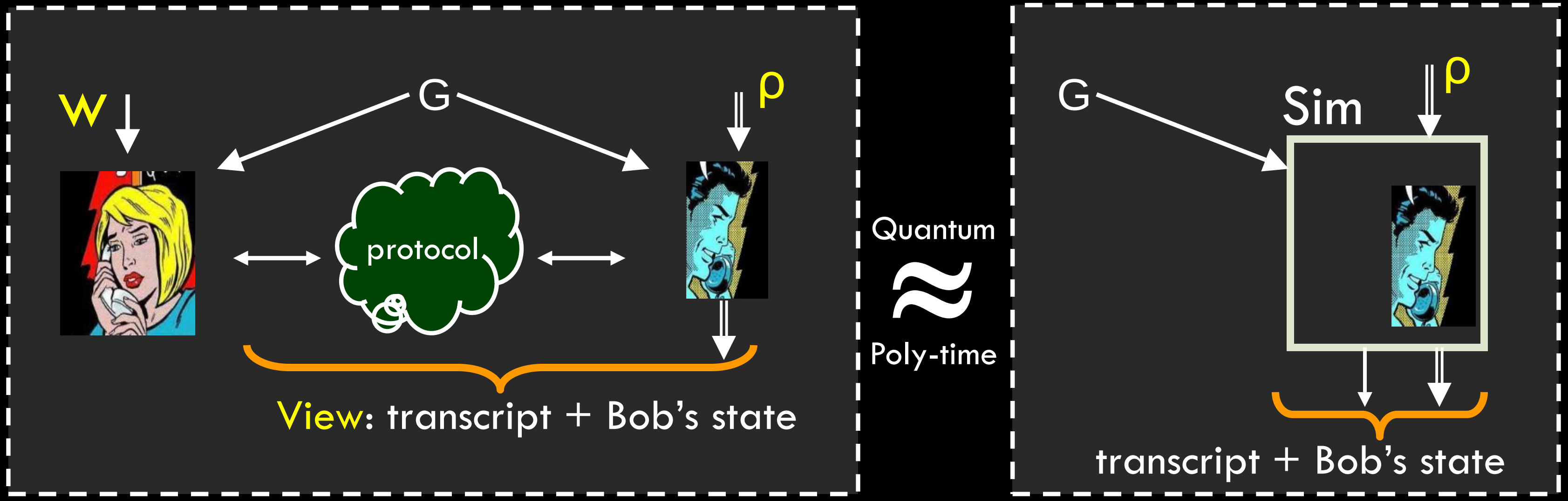
- Get around difficulty of quantum rewinding

- Revisit quantum stand-alone security models (see paper)

Formalizing Zero-Knowledge

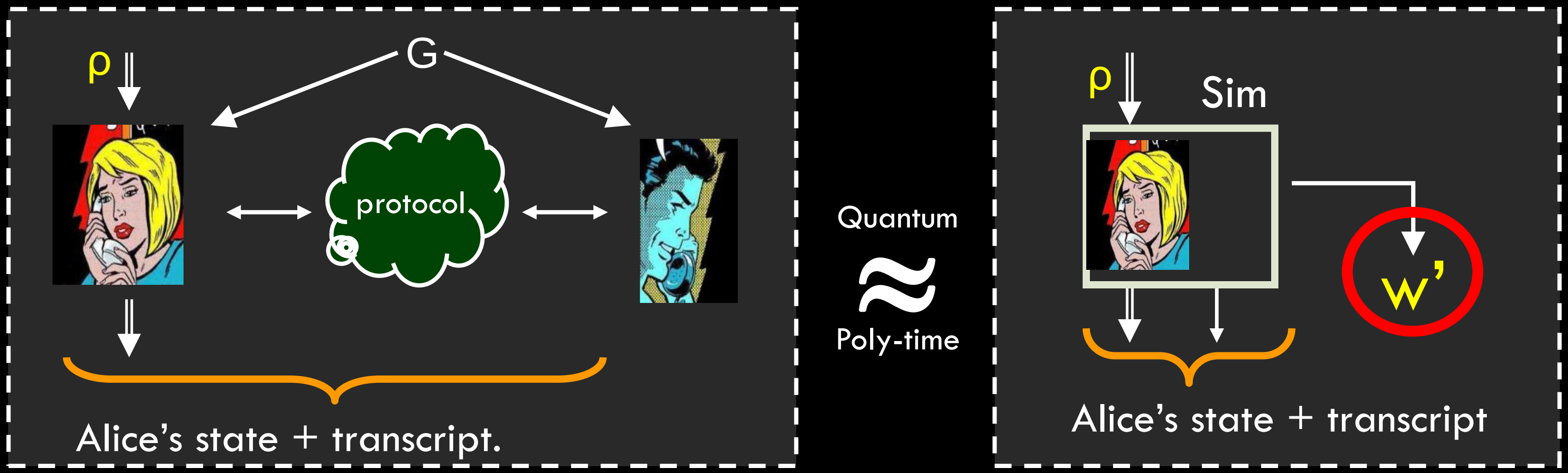
Alice wants to convince Bob graph G is 3-colorable

- **Zero knowledge:** Bob does NOT learn the coloring w
- ∀ Bob, ∃ Simulator such that ∀ quantum state ρ:



Formalizing Proofs of Knowledge

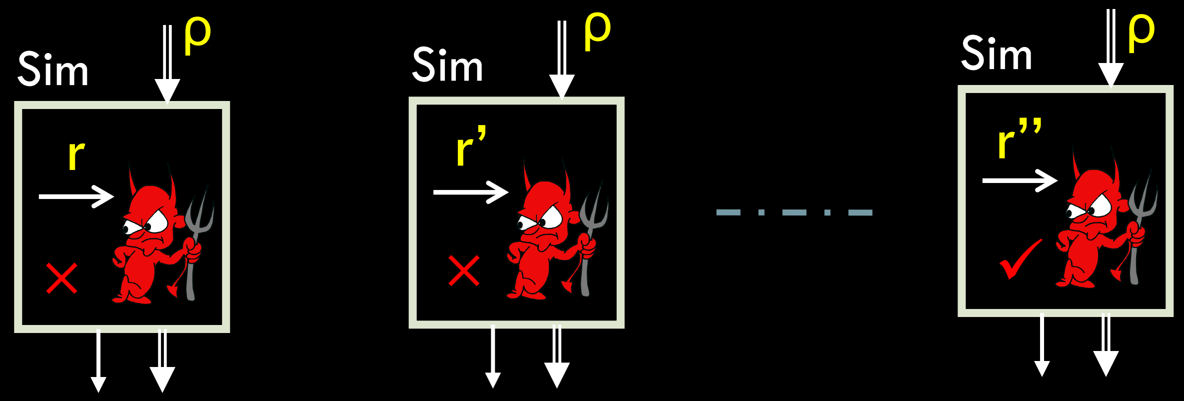
- PoK: Bob wants to be sure that Alice has some real w in mind
- ∀ Alice, ∃ Simulator such that ∀ quantum ρ



- Extra condition on simulator: if simulated transcript accepts, then extracting a 3-coloring w'of G.
  - "Witness-extended simulator"
- Fully simulatable: Simulation + Extraction

Difficulty of Quantum Rewinding

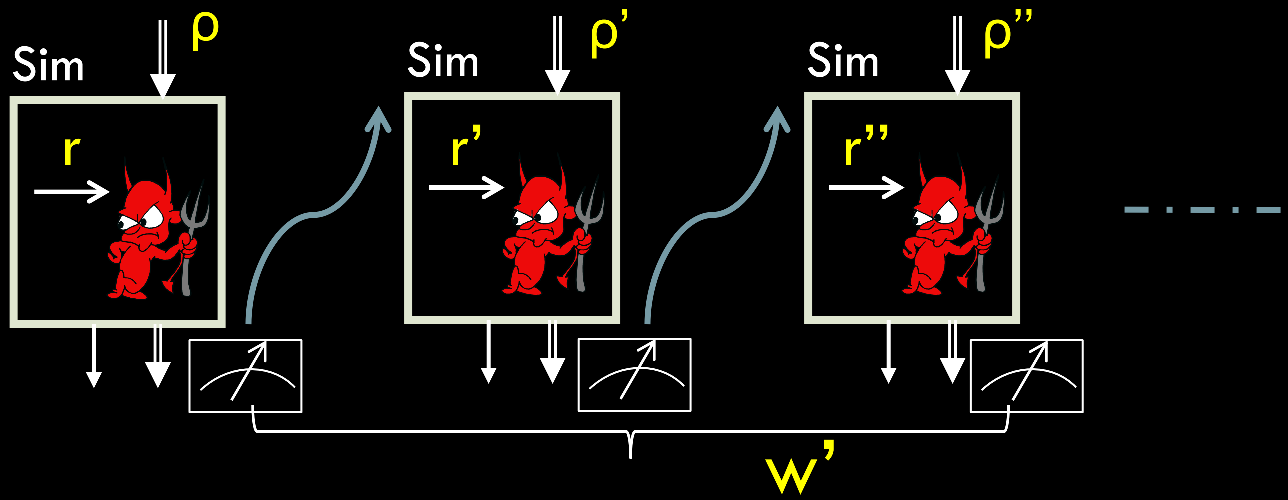
- Classical technique to construct a simulator: Rewinding
  - In every real interaction, prover answers questions from verifier
  - Without a witness, simulator may not be able to answer all questions
  - Pick a random branch from all interactions, check if could proceed
  - If NOT, "rewind" and try again from the same auxiliary input ρ



- Naïve rewinding requires taking a snapshot of the adversary's state and later returning to it
  - Quantum no-cloning!
  - Even just checking success/failure may destroy ρ

Watrous's Rewinding Technique & Limit

- Theorem [Watrous'09]: ∃ ZK proof for **NP** against quantum verifiers.
  - "Oblivious" quantum rewinding
    - If: probability of succ/failure independent of ρ
    - Then: safe to go back; but cannot remember anything

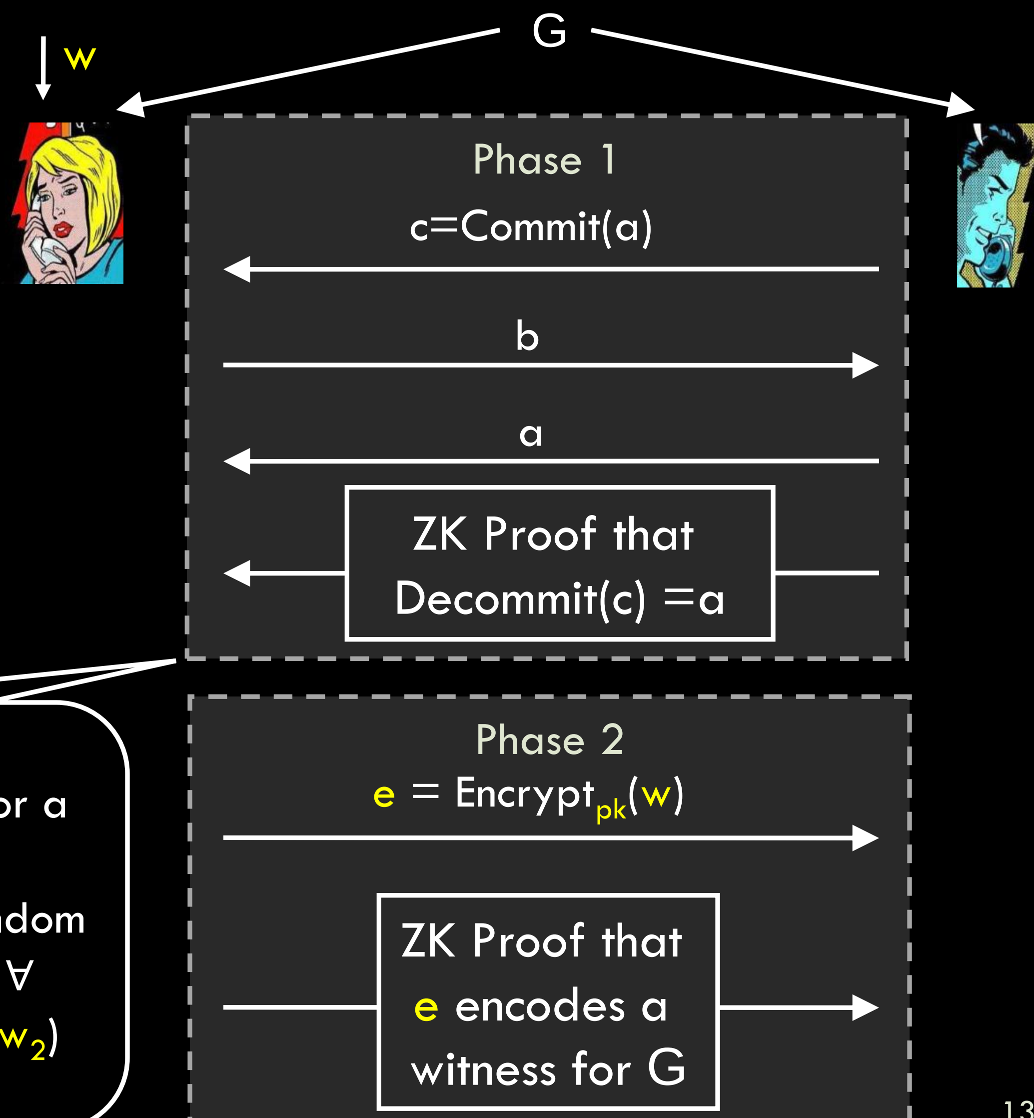


- However, NOT enough for PoK: Simulation + Extraction
  - Collecting answers from multiple branches
  - Mere extraction is possible [Unruh'10]
  - Unclear how to do **both** simultaneously

Fully Simulatable ZKPoK: Our Construction

Idea (inherited from Non-interactive ZK):

- Start with a “coin-flipping” preamble
  - Honest prover can make sure the outcome is uniformly random
  - A PoK simulator (playing the verifier) can control the outcome

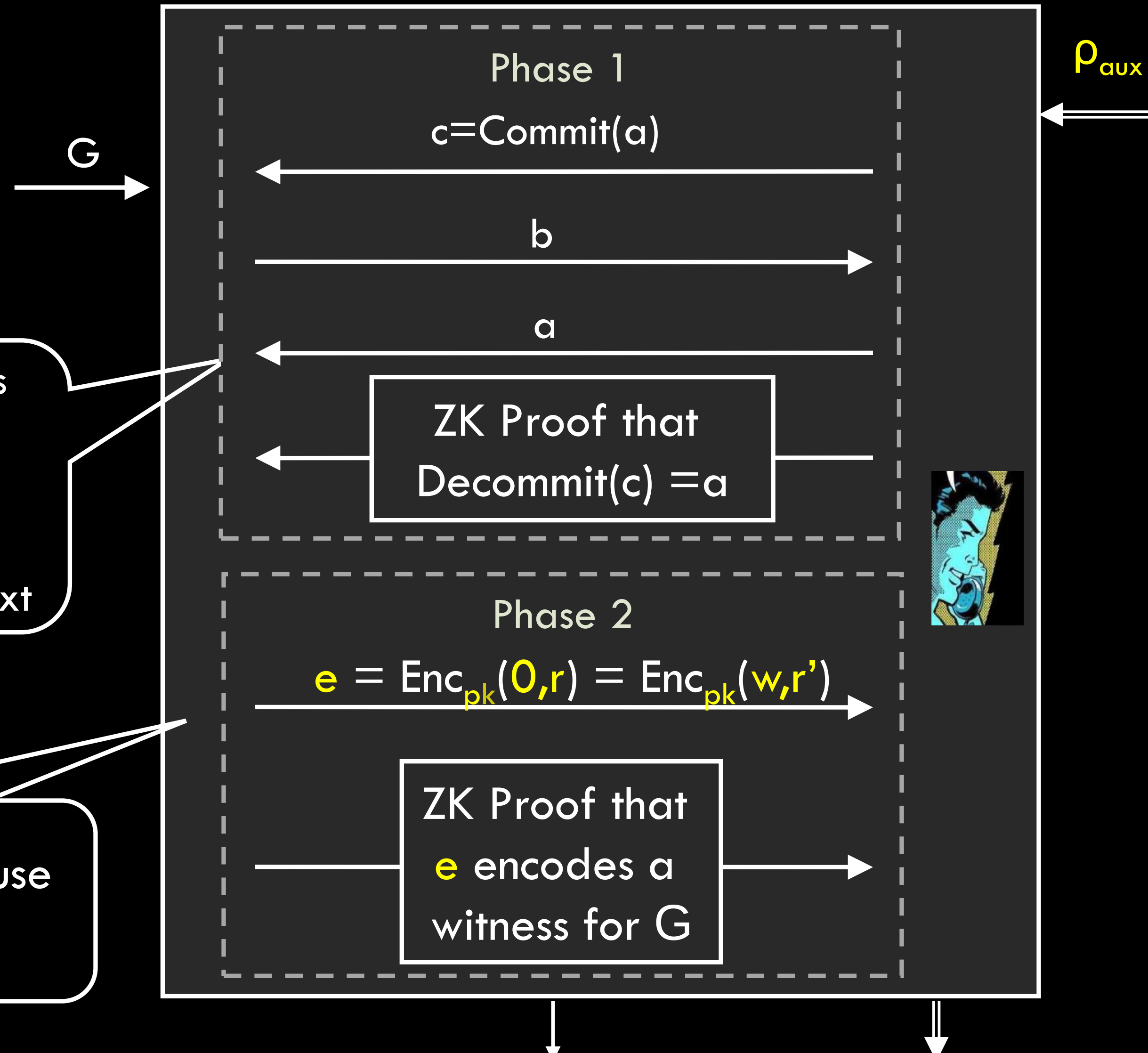


pk = a+b: interpret as public key for a special encryption scheme

- Dense: valid public key looks random
- Lossy: if pk is truly random, then $\forall$ w_1, w_2 $\text{Encrypt}_{pk}(w_1) \equiv \text{Encrypt}_{pk}(w_2)$

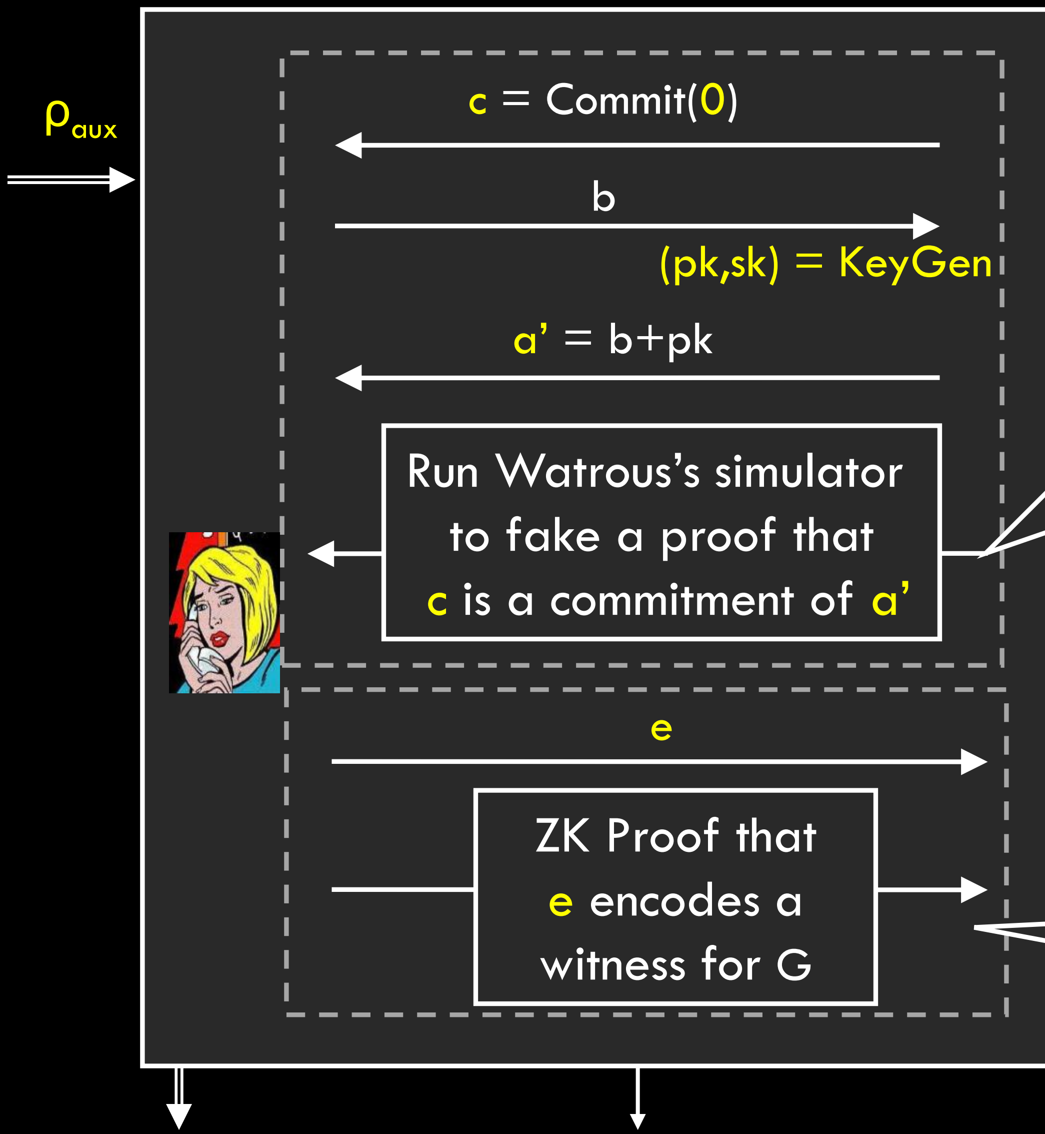
ZK: simulating dishonest verifiers

- The outcome of Phase 1 is pk : a truly random string
- Under such a key, any ciphertext can be "decoded" to any plaintext



Simulator succ. w.h.p because input is a true instance

PoK: simulating dishonest provers



- Simulator succ. w.h.p. because comm(0) is indistinguishable from comm(a')
- If succ., coin flipping will be a'+b = pk, of which the simulator knows the decrypting key

$w' = Dec_{sk}(e)$

Soundness ensures that: if ZK succ, w' is a valid witness for G w.h.p.

Putting It All Together

- Recap:
  - Fully simulatable ZKPoK ⇨ quantum-secure SFE
  - ∃ Fully simulatable ZKPoK Protocol
- Corollary1: Modular composition ⇨ Quantum-secure SFE in plain model (i.e., no trusted set-up) assuming quantum-secure
  - dense & lossy encryption
  - pseudorandom generator
- Corollary2: An interesting equivalence: CF = ZKPoK
  - Round-complexity preserving reductions
- Independent Work [Lunemann,Nielsen'11]
  - Fully simulatable quantum-secure coin-flipping
  - Plug into [GMW'87] and obtain similar feasibility results as ours
- What I didn't talk about our work: Models, UC-security etc. (see paper)

Conclusion

- Some key pieces of classical crypto unchanged in presence of quantum attackers
- A lot more remains unclear…

- Open Questions:
  - Can we extend to other settings: e.g., multi-party and concurrent security?
  - Round complexity: ∃ quantum-secure constant round ZK/CF?
  - Is there any natural two-party classical protocol that is broken by quantum adv's **NOT** because of computational assumptions?

Thank you!

Reference

- [BB’84] C.H. Bennett, G. Brassard "Quantum cryptography: Public-key distribution and coin tossing". Proceedings of IEEE International Conference on Computers, Systems and Signal Processing 1984.
- [BM'05] Michael Ben-Or, Dominic Mayers. “General Security Definition and Composability for Quantum & Classical Protocols”. quant-ph/0409062.
- [C’00] Ran Canetti. “Security and Composition of Multiparty Cryptographic Protocols”. J. Cryptology. 2000.
- [CF’01] Ran Canetti, Marc Fischlin. “Universally Composable Commitments”. Crypto 2001.
- [CLOS’02] Ran Canetti, Yehuda Lindell, Rafail Ostrovsky, and Amit Sahai, “Universally composable two-party and multi-party secure computation”. STOC 2002, pp. 494–503.
- [CSST'05] C. Crepeau, Louis Salvail J.-R. Simard, A. Tapp. “Classical and quantum strategies for two-prover bit commitments”. Manuscript 2005.
- [DL’09] Ivan Damgård, Carolin Lunemann. “Quantum-Secure Coin-Flipping and Applications”. ASIACRYPT 2009.
- [FS’09] Serge Fehr, Christian Schaffner. “Composing Quantum Protocols in a Classical Environment”. TCC 2009.
- [LC’98] H.-K. Lo, H. F. Chau. “Why Quantum Bit Commitment And Ideal Quantum Coin Tossing Are Impossible”. Physica D120 (1998) 177-187. quant-ph/9711065.
- [LC99] Hoi-Kwong Lo, H. F. Chau. “Unconditional Security of Quantum Key Distribution over Arbitrarily Long Distances”. Science 26 March 1999: Vol. 283. no. 5410, pp. 2050 - 2056
- [M’97] D. Mayers. “Unconditonally secure quantum bit commitment is impossible”. Phys. Rev. Lett. 78, (1997) 3414-3417.
- [S'94] Peter W. Shor. “Algorithms for Quantum Computation: Discrete Logarithms and Factoring” FOCS 1994: 124-134.
- [W'09] J. Watrous. “Zero-knowledge against quantum attacks”. J. on Computing, 2009.
- [U’10a] Dominique Unruh. “Quantum proofs of knowledge” April 2010, Preprint on IACR ePrint 2010/212.
- [U’10b] Dominique Unruh. “Universally composable quantum multi-party computation”. EUROCRYPT 2010